A Unified Approach to Differentially Private Bayes Point Estimation

Braghadeesh Lakshminarayanan and Cristian R. Rojas

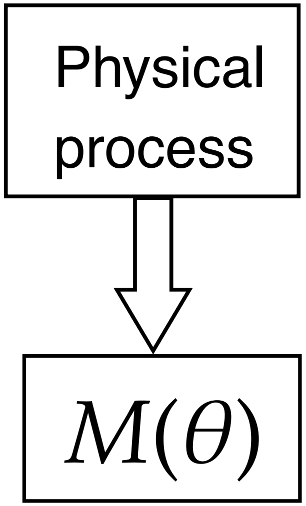
Division of Decision and Control Systems
KTH Royal Institute of Technology
Stockholm, Sweden

22nd IFAC WC, Yokohama
July 13, 2023

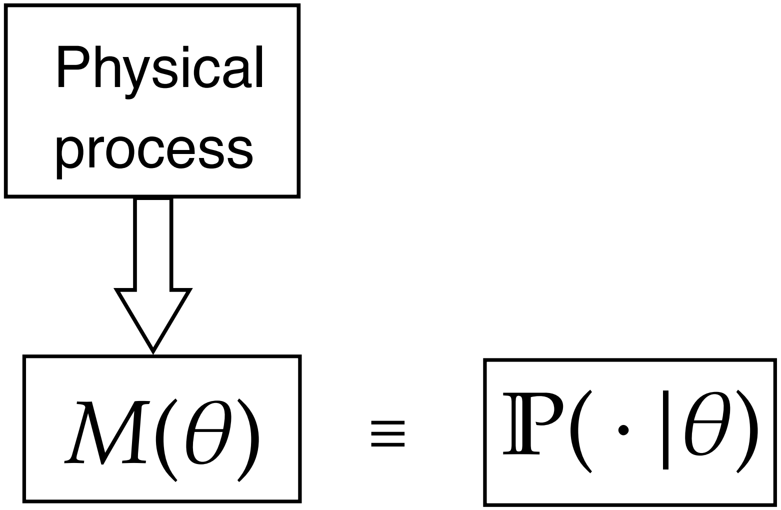
Motivation and Background

I. Point Estimate

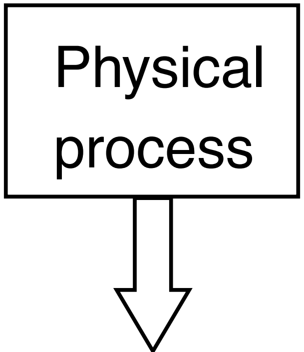
Point Estimate



Point Estimate

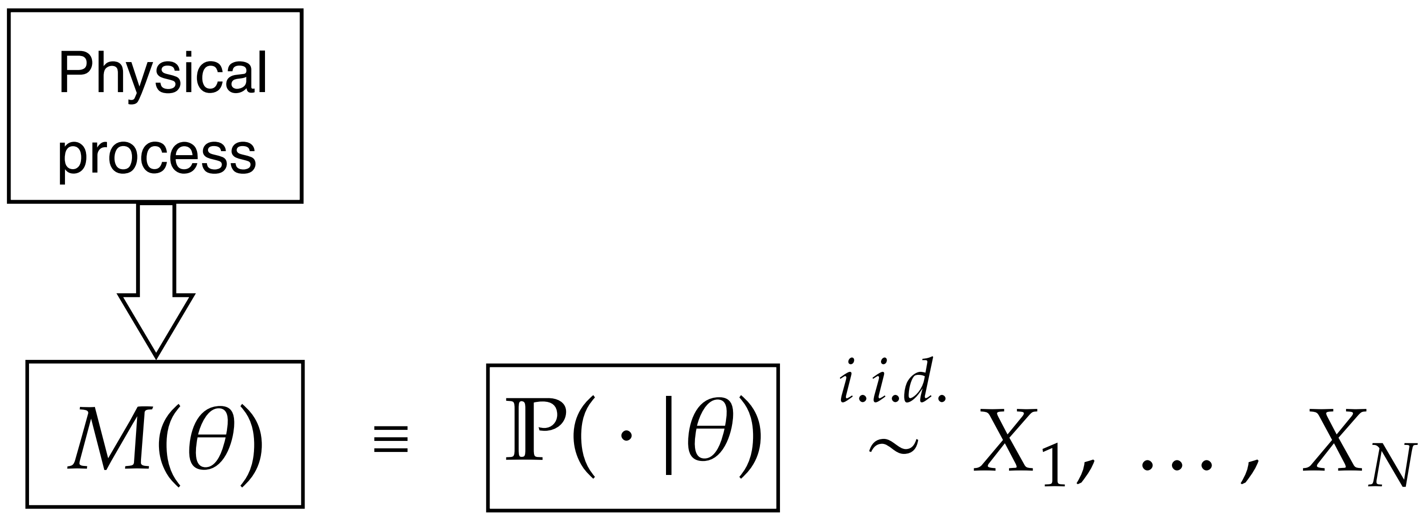


Point Estimate



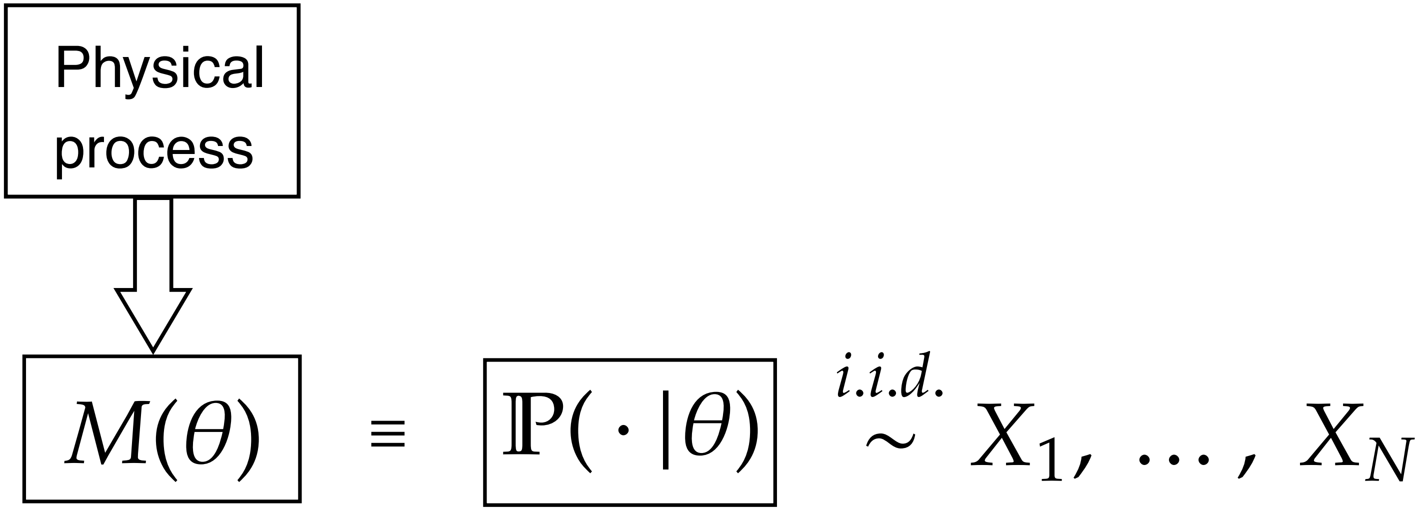
$$M(\theta) \equiv \mathbb{P}(\,\cdot\,|\theta) \overset{i.i.d.}{\sim} X_1, \ldots, X_N$$

Point Estimate



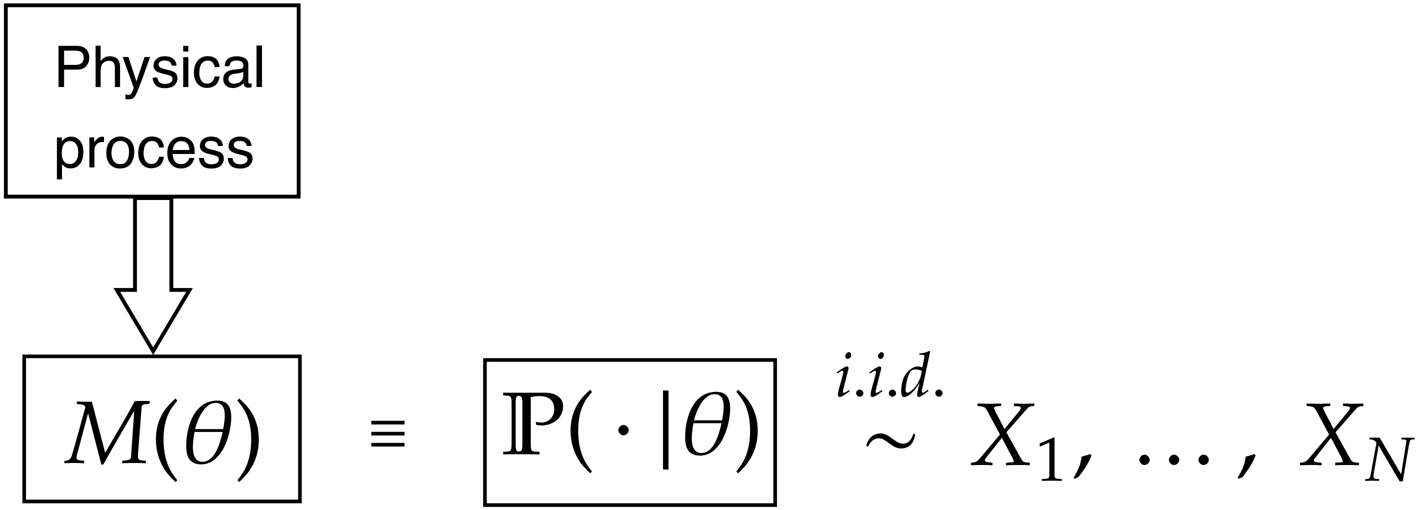
- Goal: Estimate unknown θ by observing $X = (X_1, \ldots, X_N)$

Point Estimate



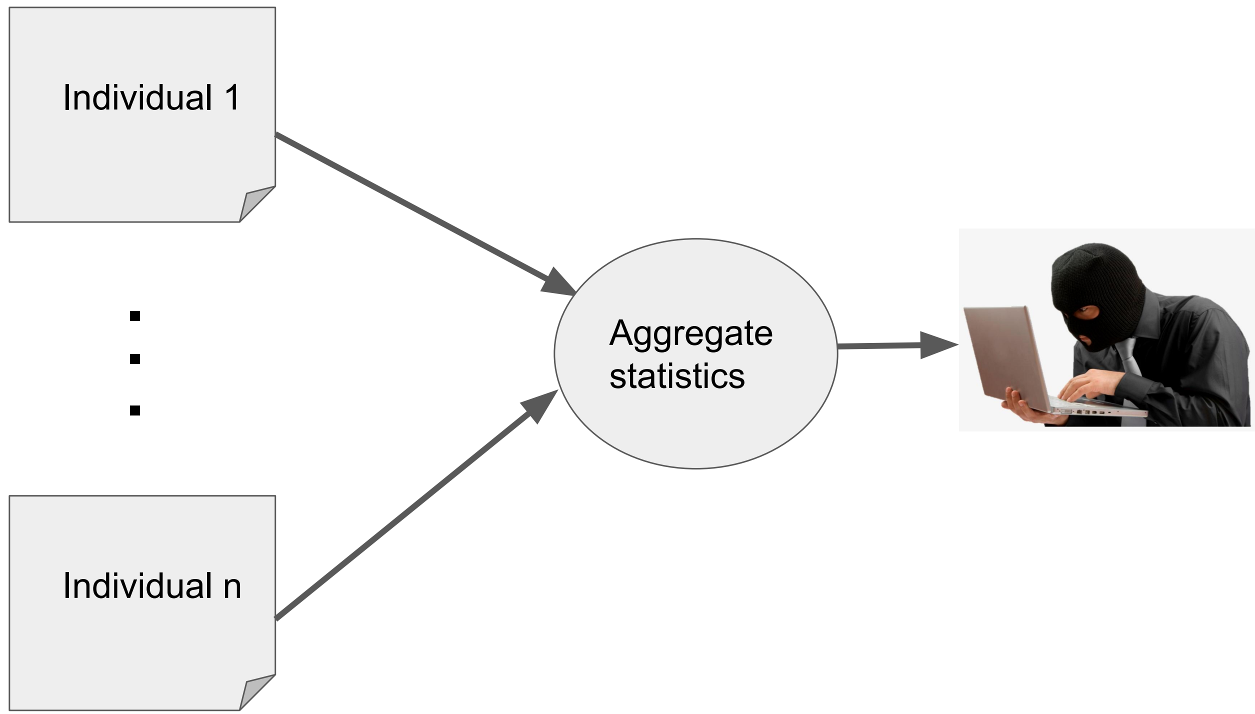
- Goal: Estimate unknown θ by observing $X = (X_1, \ldots, X_N)$
- Point estimate: $\hat{\theta} := \hat{\theta}(X)$ Single quantity that is a possible value of θ

Point Estimate



- Goal: Estimate unknown θ by observing $X = (X_1, \ldots, X_N)$
- Point estimate: $\hat{\theta} := \hat{\theta}(X)$ Single quantity that is a possible value of θ
- Examples:
  - Ber(θ): $\hat{\theta} = \frac{1}{N}\sum_{i=1}^{N} X_i$
  - $\mathcal{N}(\mu, \sigma^2)$: $\widehat{\sigma^2} = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \mu)^2$

Need for Privacy in Point Estimates

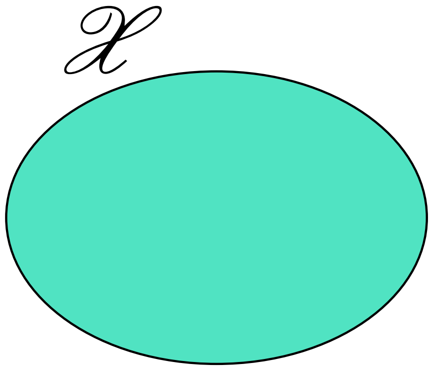


- Aggregate statistics: Sample mean, sample covariance,...
- Possible to infer an individual[1]

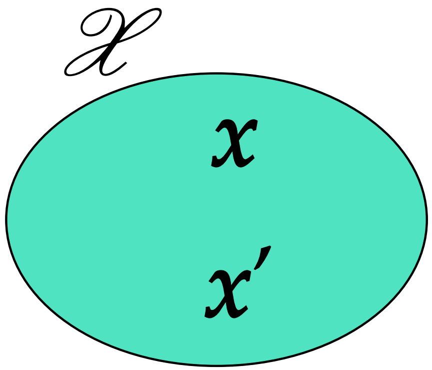
[1] Homer, N. et al. “Resolving individuals contributing trace amounts of DNA to highly complex mixtures using high-density SNP genotyping microarrays”. PLOS Genetics, 2008.

II. Differential Privacy

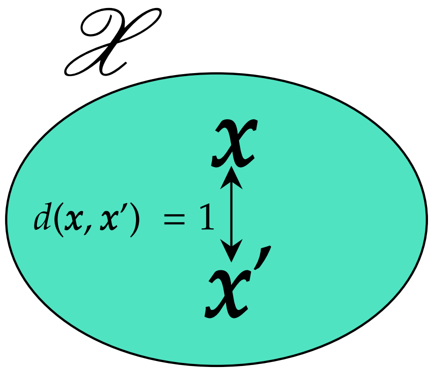
Differential Privacy (DP)



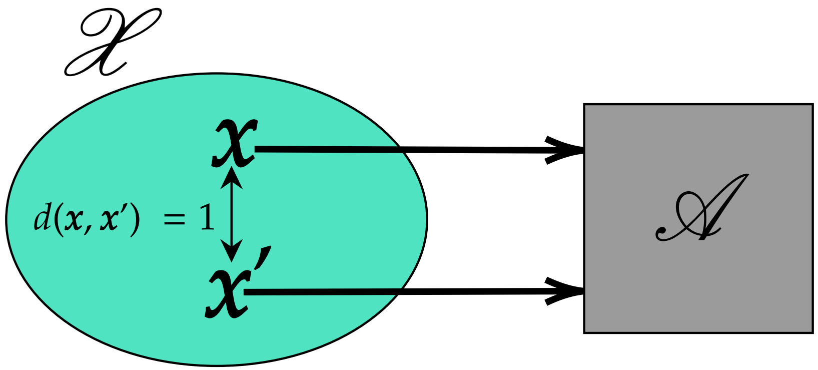
Differential Privacy (DP)



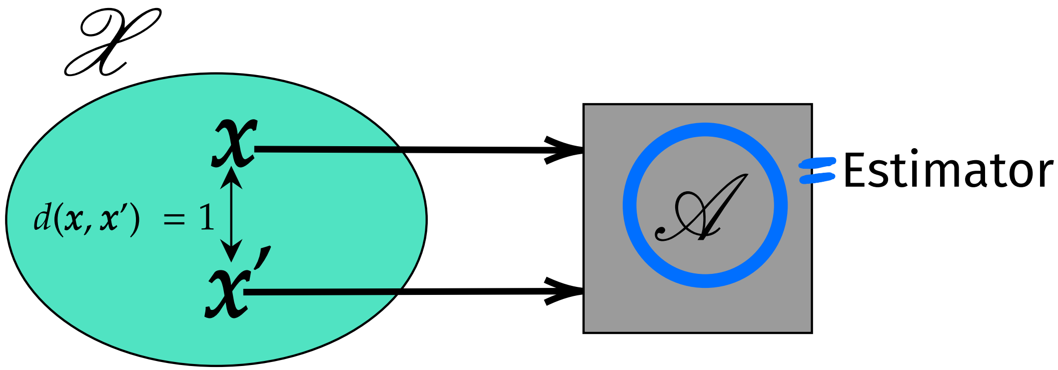
Differential Privacy (DP)



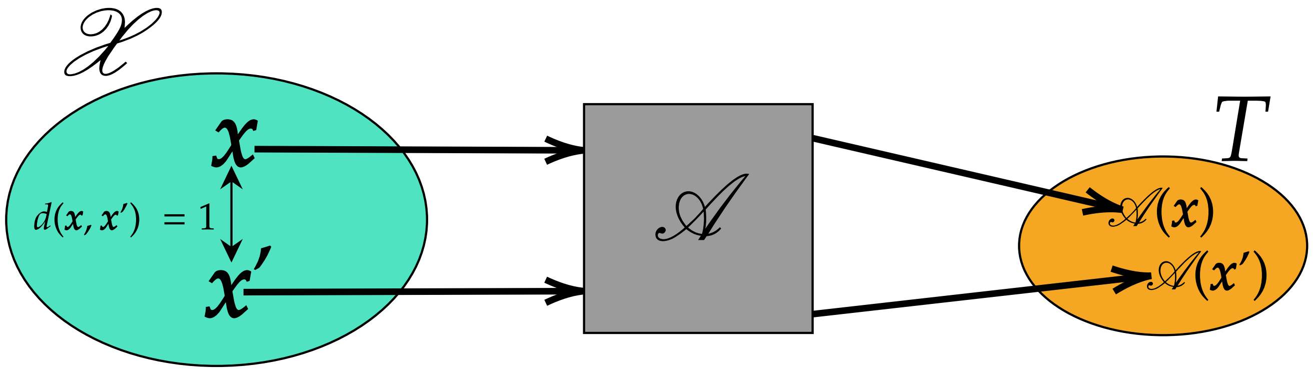
Differential Privacy (DP)



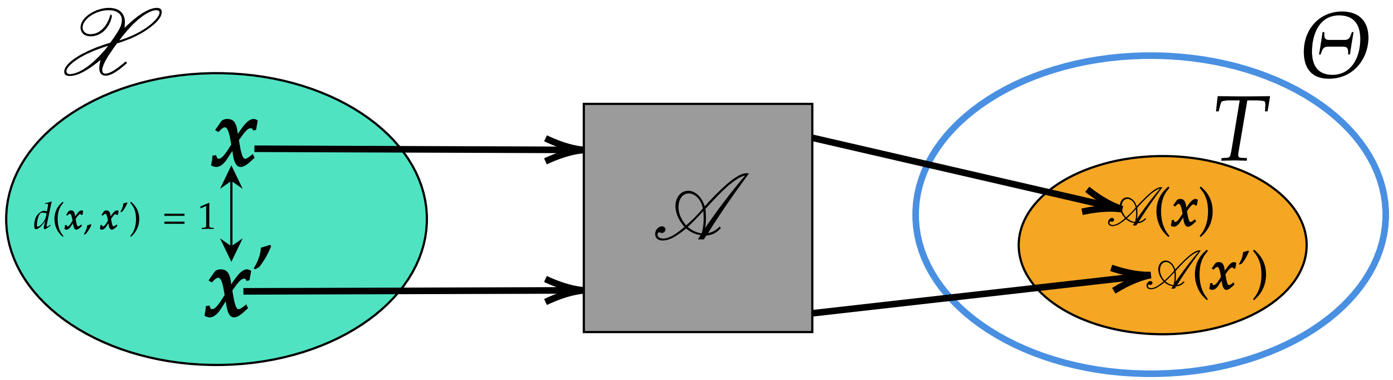
Differential Privacy (DP)



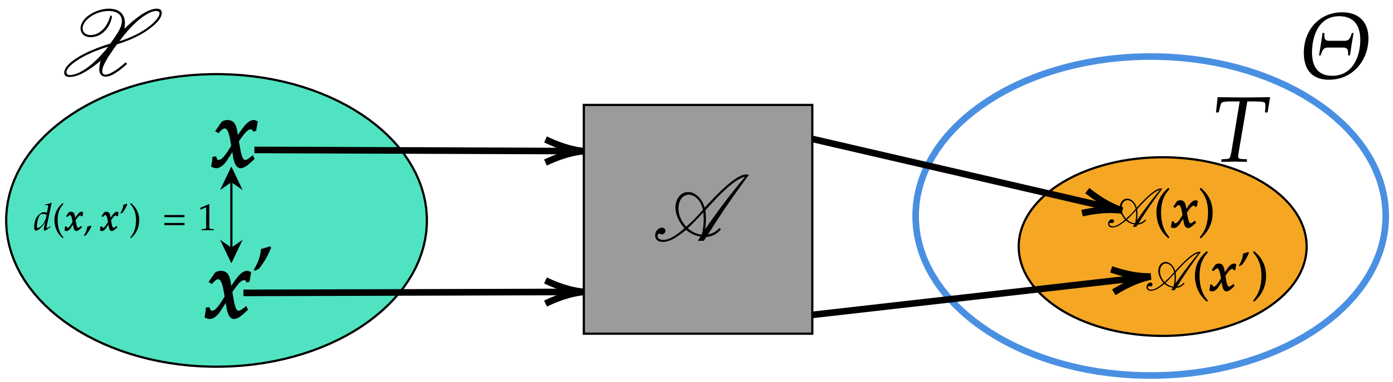
Differential Privacy (DP)



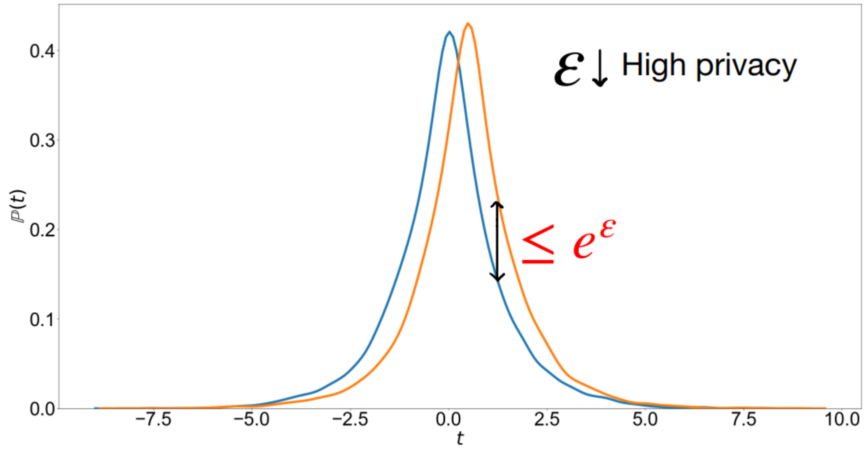
Differential Privacy (DP)



Differential Privacy (DP)

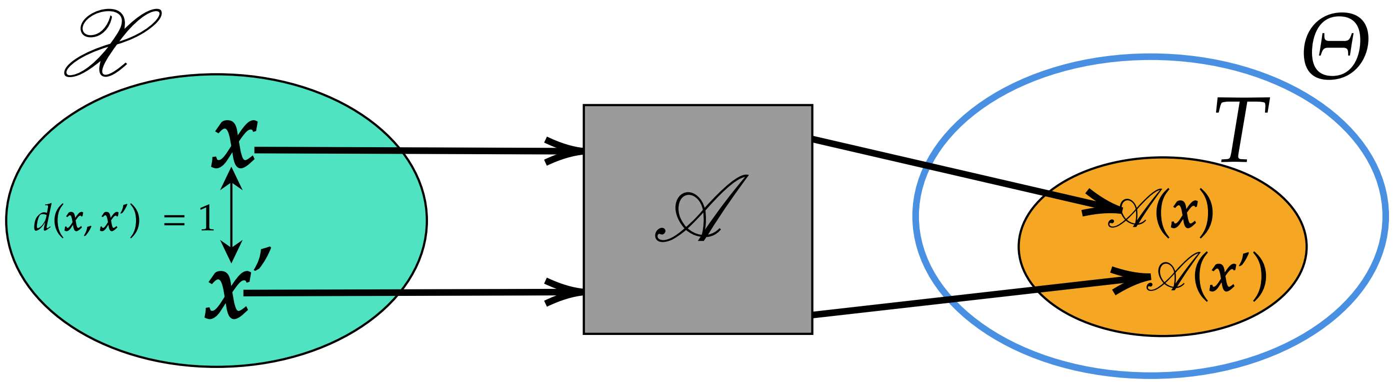


DP definition[2]: $\Pr[\mathscr{A}(\boldsymbol{x}) \in T] \leq e^{\varepsilon} \Pr[\mathscr{A}(\boldsymbol{x}') \in T]$

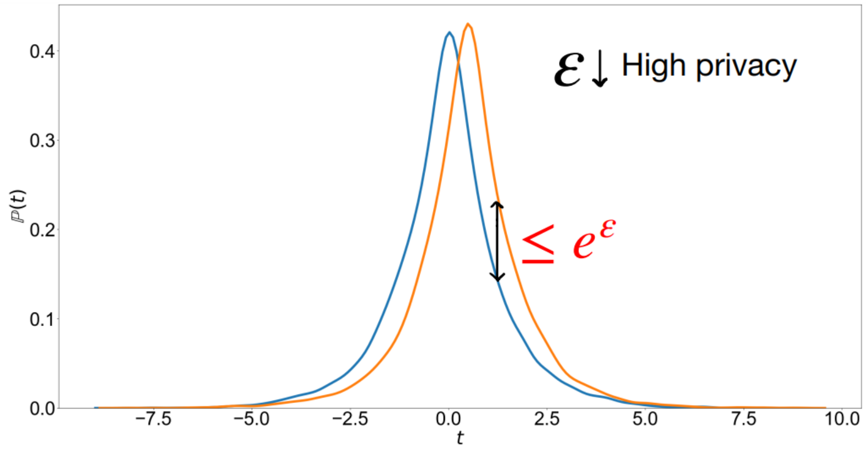


[2] C. Dwork and A. Roth. "The Algorithmic Foundations of Differential Privacy". Foundations and Trends in Theoretical Computer Science. 2014

Differential Privacy (DP)

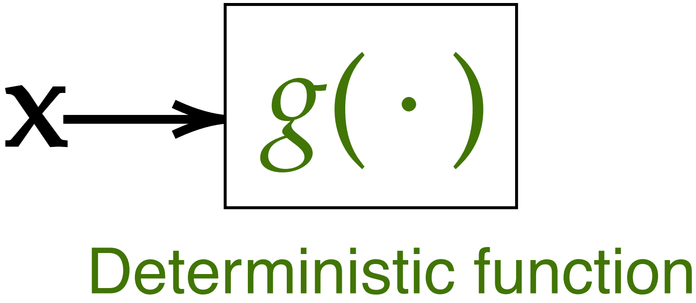


DP definition: $\Pr[\mathscr{A}(\boldsymbol{x}) \in T] \leq e^{\varepsilon} \Pr[\mathscr{A}(\boldsymbol{x}') \in T]$

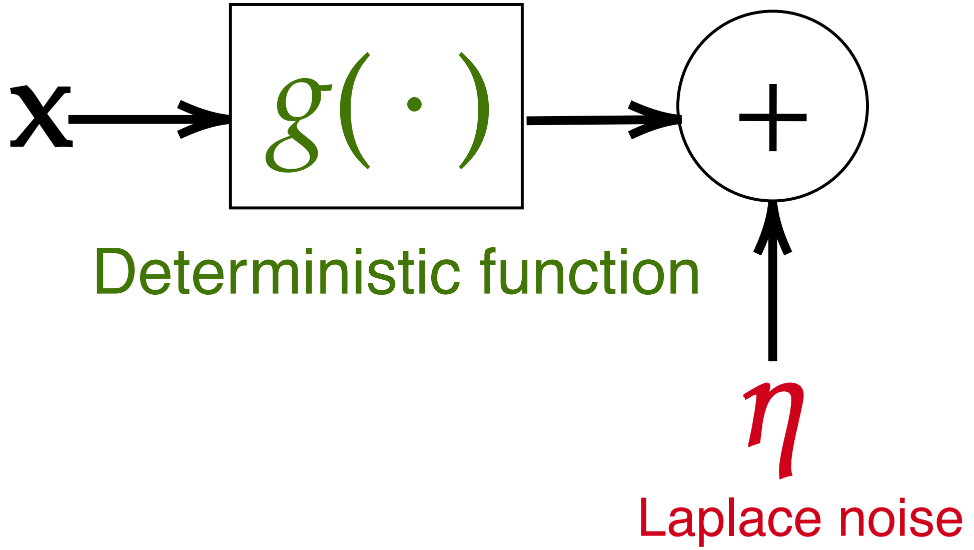


How to design $\mathscr{A}$?

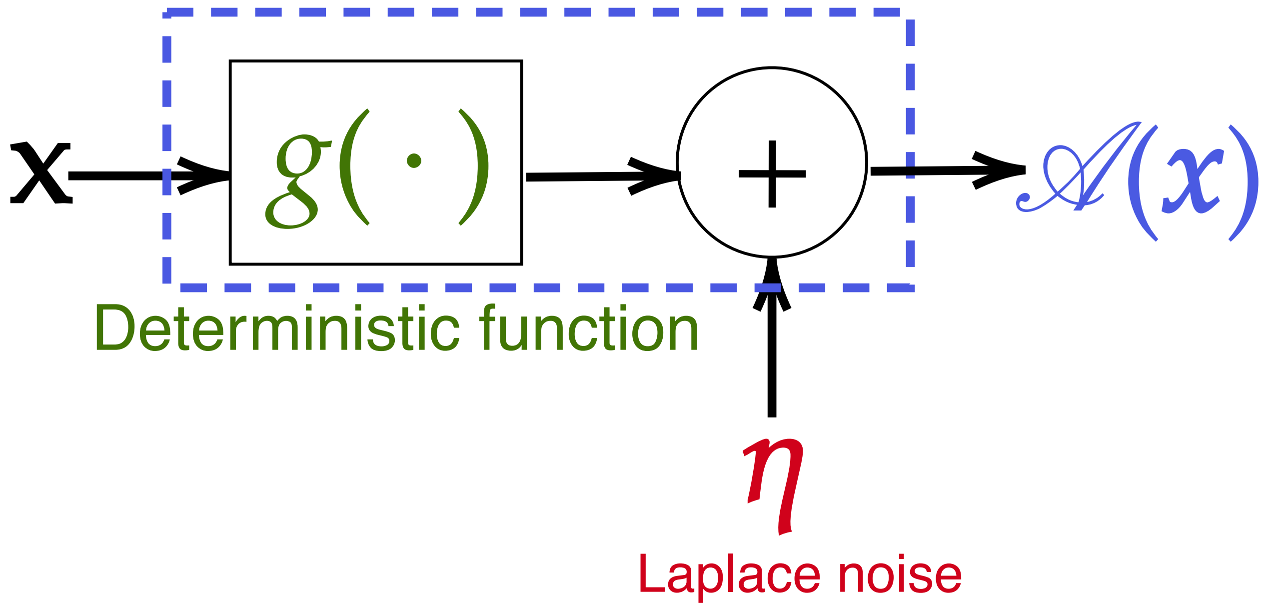
The Laplace Mechanism



The Laplace Mechanism

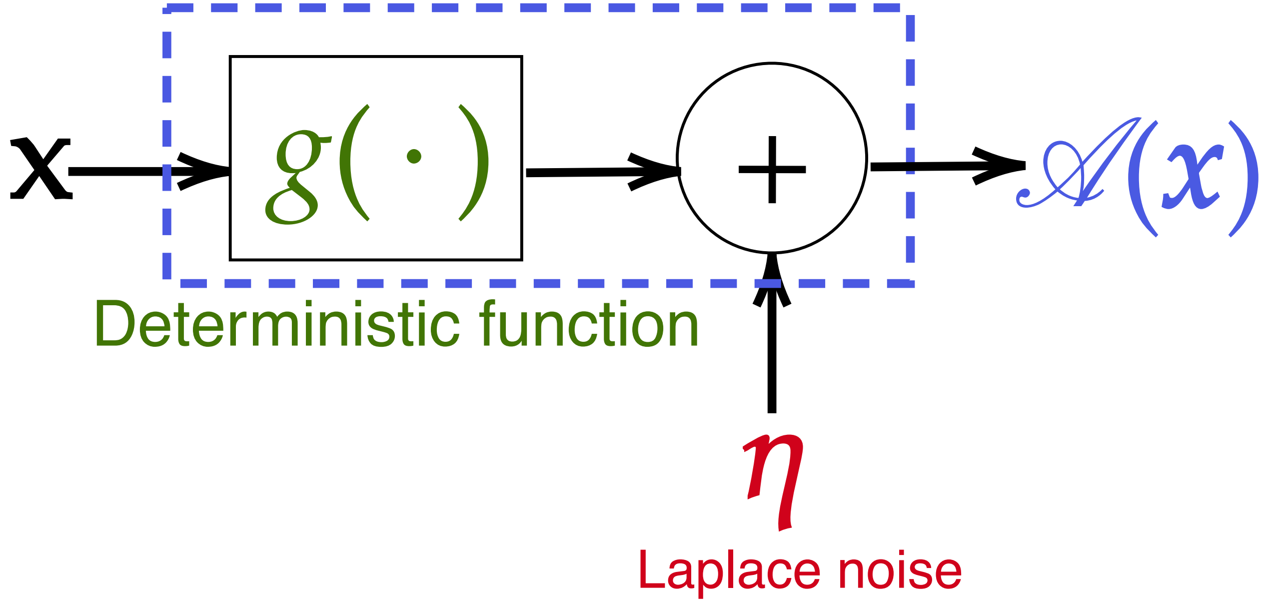


The Laplace Mechanism



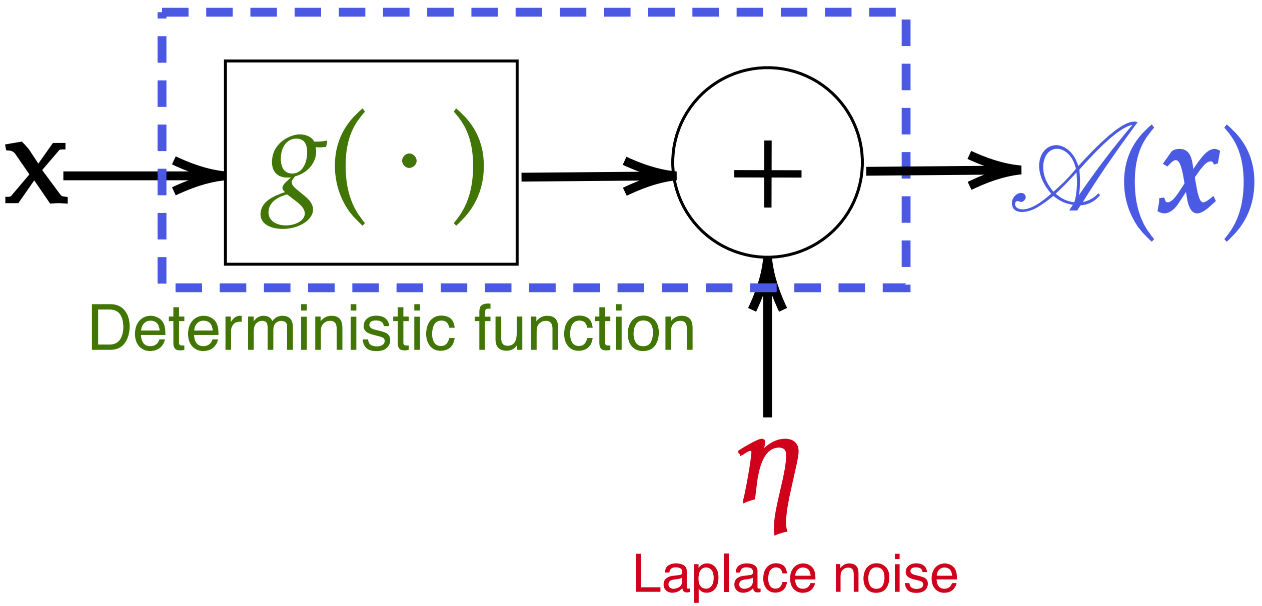
$\eta_i \sim \text{Lap}(0, \frac{\sigma_g}{\varepsilon})$

The Laplace Mechanism



$\eta_i \sim \text{Lap}(0, \frac{\sigma_g}{\varepsilon})$ =?

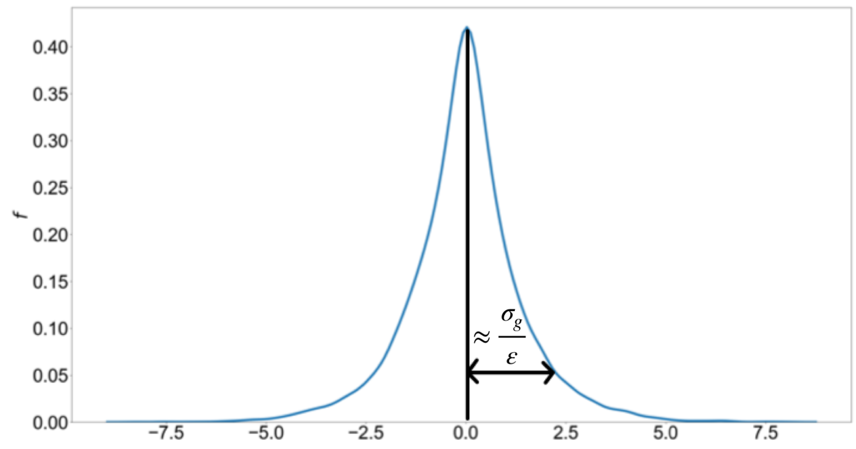
The Laplace Mechanism



$\eta_i \sim \text{Lap}(0, \frac{\sigma_g}{\varepsilon})$

$$\boxed{\sigma_g = \sup_{\mathbf{x},\mathbf{x}' \in \mathcal{X}: d(\mathbf{x},\mathbf{x}')=1} \|g(\mathbf{x}) - g(\mathbf{x}')\|_1}$$ l_1 sensitivity

The Laplace Mechanism

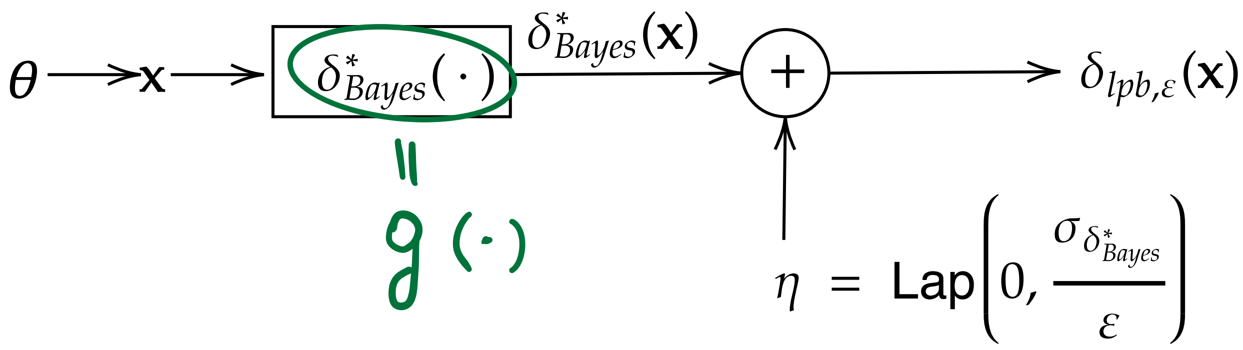


- Laplace mechanism enforces DP[3]

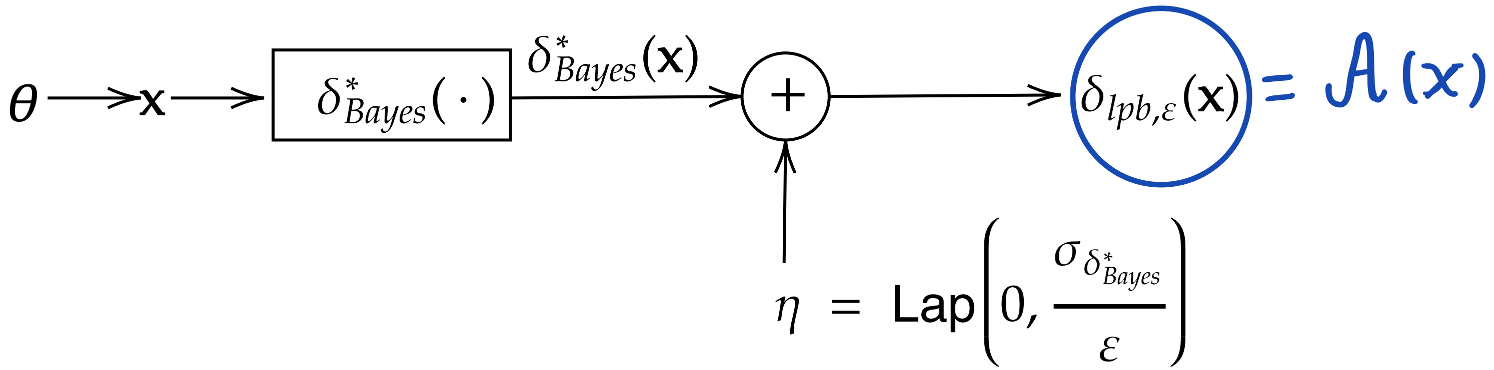
- DP via Laplace mechanism encounters accuracy-privacy trade off

[3] C. Dwork and A. Roth. "The Algorithmic Foundations of Differential Privacy". Foundations and Trends in Theoretical Computer Science. 2014

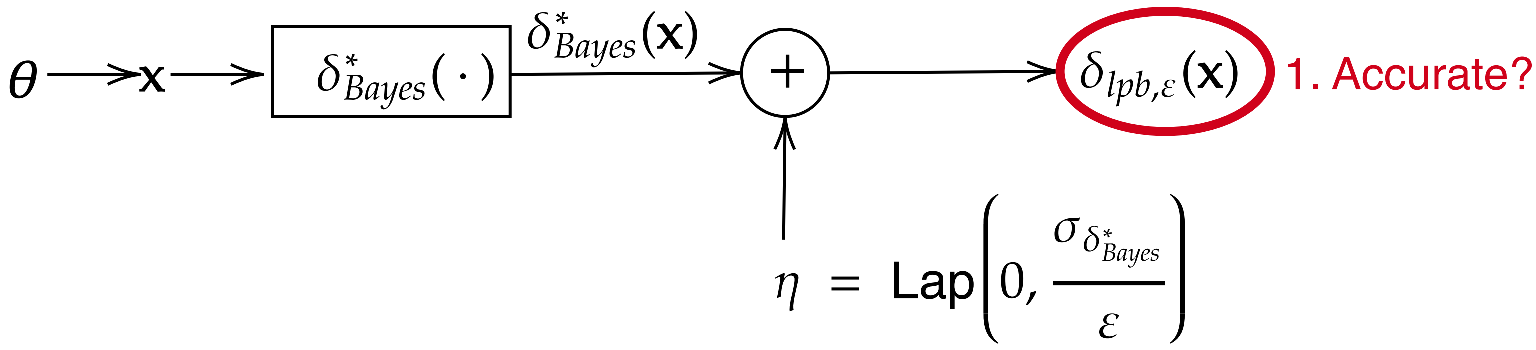
Bayes Point Estimate + Differential Privacy



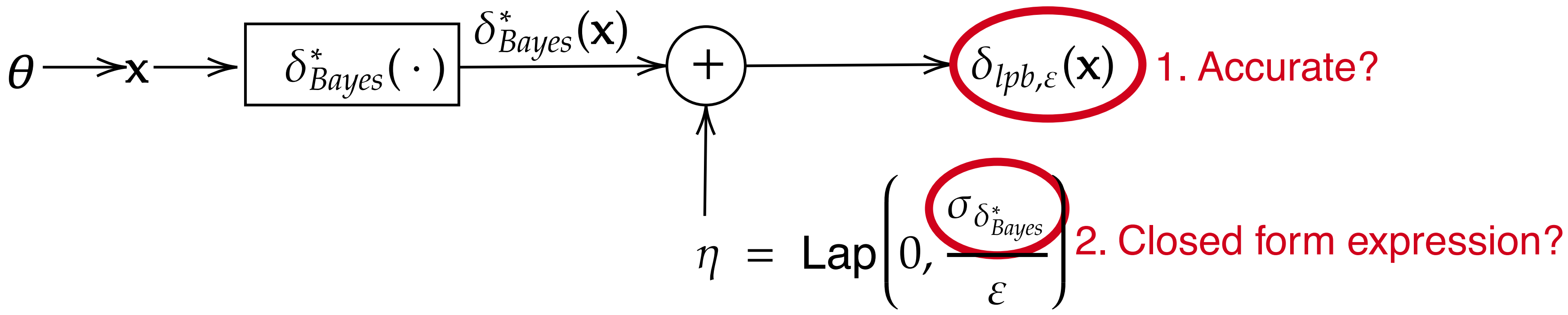
Bayes Point Estimate + Differential Privacy



Bayes Point Estimate + Differential Privacy



Bayes Point Estimate + Differential Privacy



Outline

Unified Approach (UBaPP Estimator)

UBaPP Estimator for Finite Case

Numerical Example

Conclusion

Outline

Unified Approach (UBaPP Estimator)

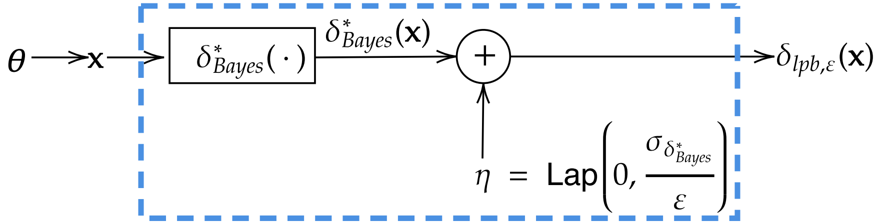
UBaPP Estimator for Finite Case

Numerical Example

Conclusion

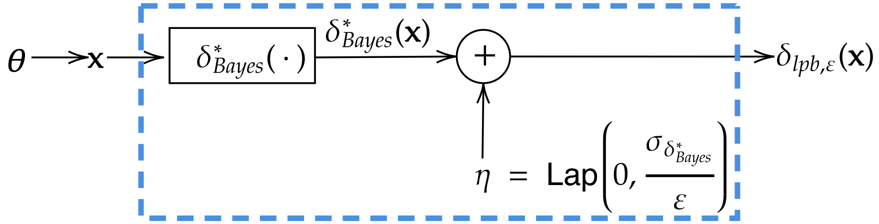
Unified Approach (UBaPP Estimator)

Earlier,

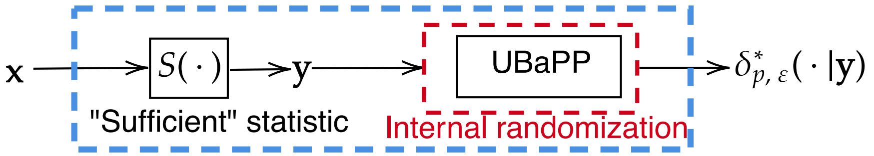


Unified Approach (UBaPP Estimator)

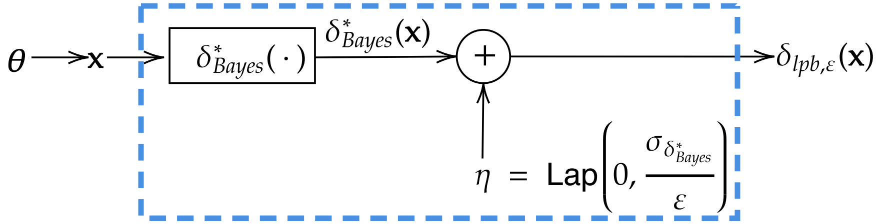
Earlier,



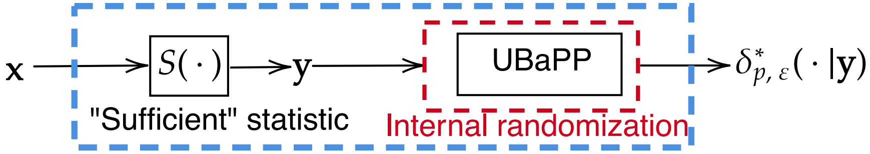
Instead, we propose



Earlier,



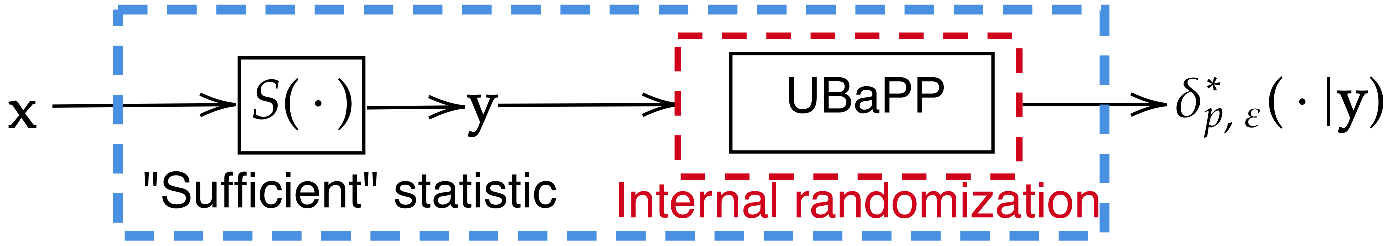
Instead, we propose



Randomized estimator (our approach)

DP is enforced by randomizing the estimator directly

Unified Approach (UBaPP Estimator)

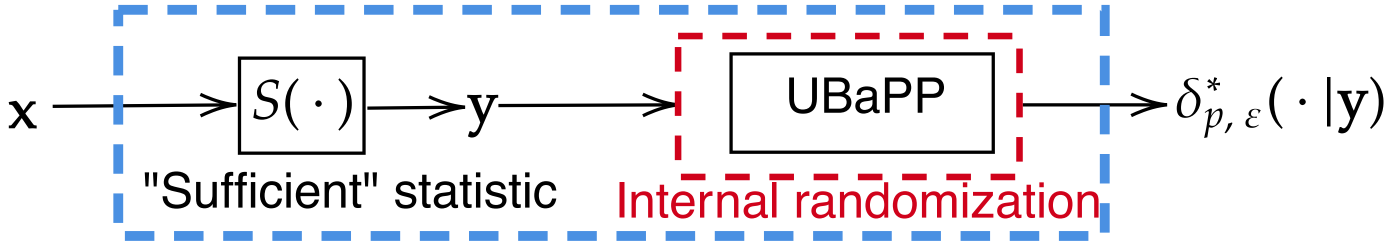


Non-private Bayes risk minimization:

Minimize risk

$$R(\delta, \pi) = \int_{\theta \in \Theta} \int_{\mathbf{y} \in \mathcal{Y}} L(\theta, \delta) q_\theta(\mathbf{y}) \pi(\theta) d\theta d\mathbf{y}$$

Unified Approach (UBaPP Estimator)



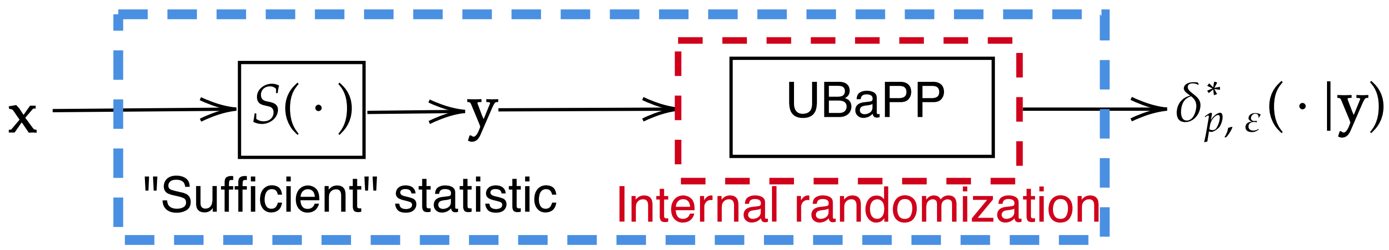
<u>Non-private Bayes risk minimization:</u>

Minimize risk

$$R(\delta, \pi) = \int_{\theta \in \Theta} \int_{\mathbf{y} \in \mathcal{Y}} L(\theta, \delta) q_\theta(\mathbf{y}) \pi(\theta) d\theta d\mathbf{y}$$

Solution: Deterministic estimate!

Unified Approach (UBaPP Estimator)



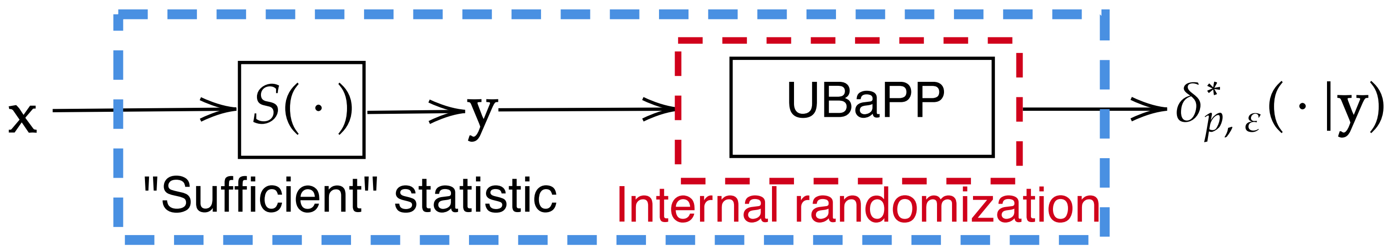
Private-Bayes risk minimization:

Minimize randomized risk

$$R\left(\delta_{p,\varepsilon}, \pi\right) = \int_{\theta\in\Theta}\int_{\mathbf{y}\in\mathcal{Y}}\int_{\tilde{\theta}\in\Theta} L(\theta, \tilde{\theta}) q_{\theta}(\mathbf{y})\pi(\theta)d\theta d\mathbf{y}\delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y})d\tilde{\theta}$$

Solution: Randomized estimate

Unified Approach (UBaPP Estimator)



Private-Bayes risk minimization:

Minimize randomized risk

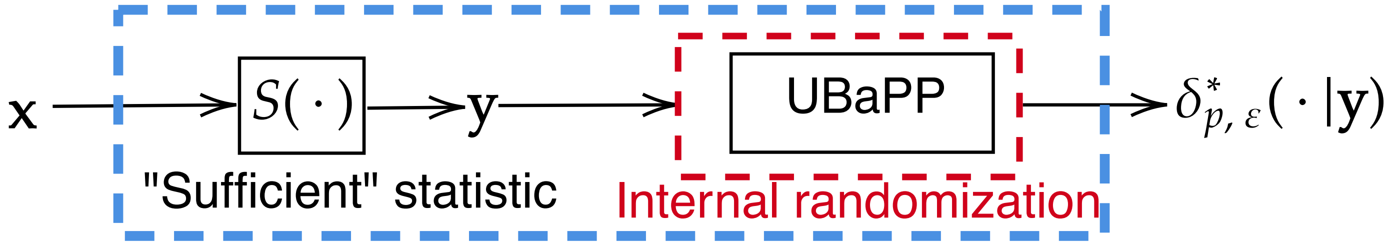
$$R\left(\delta_{p,\varepsilon}, \pi\right) = \int_{\theta\in\Theta}\int_{\mathbf{y}\in\mathcal{Y}}\int_{\tilde{\theta}\in\Theta} L(\theta,\tilde{\theta})q_\theta(\mathbf{y})\pi(\theta)d\theta d\mathbf{y}\delta_{p,\varepsilon}(\tilde{\theta}\mid\mathbf{y})d\tilde{\theta}$$

subject to

$$\delta_{p,\varepsilon}(\tilde{\theta}\mid\mathbf{y}) \le e^{\varepsilon}\delta_{p,\varepsilon}\left(\tilde{\theta}\mid\mathbf{y}'\right), \text{ for each } \tilde{\theta}\in\Theta$$

DP constraint

Unified Approach (UBaPP Estimator)



Solution: UBaPP estimate

Private-Bayes risk minimization:

Minimize randomized risk

$$R\left(\delta_{p,\varepsilon}, \pi\right) = \int_{\theta\in\Theta}\int_{\mathbf{y}\in\mathcal{Y}}\int_{\tilde{\theta}\in\Theta} L(\theta,\tilde{\theta}) q_\theta(\mathbf{y})\pi(\theta) d\theta d\mathbf{y} \delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) d\tilde{\theta}$$

subject to

$$\delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) \leq e^{\varepsilon}\delta_{p,\varepsilon}\left(\tilde{\theta} \mid \mathbf{y}'\right), \text{ for each } \tilde{\theta} \in \Theta$$

DP constraint

Unified Approach (UBaPP Estimator)

UBaPP estimator is the solution to following convex program:

$$\min_{\delta_{p,\varepsilon} \in \mathcal{P}(\mathcal{Y},\Theta)} \int_{\Theta} \int_{\mathcal{Y}} \int_{\Theta} L(\theta, \tilde{\theta}) \delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) q_{\theta}(\mathbf{y}) \pi(\theta) d\theta d\mathbf{y} d\tilde{\theta}$$

$$\text{s.t.} \quad \delta_{p,\varepsilon}(\tilde{\theta} \mid S(\mathbf{x})) \leq e^{\varepsilon} \delta_{p,\varepsilon}\left(\tilde{\theta} \mid S(\mathbf{x}')\right), \text{ for each } \tilde{\theta} \in \Theta$$

$$\text{and } \mathbf{x}, \mathbf{x}' \in \mathcal{X} \text{ s.t. } d(\mathbf{x}, \mathbf{x}') = 1$$

$$\int_{\Theta} \delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) d\mathbf{y} = 1, \text{ for each } \mathbf{y} \in \mathcal{Y}$$

$$\delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) \geq 0, \text{ for each } \mathbf{y} \in \mathcal{Y}, \tilde{\theta} \in \Theta$$

Unified Approach (UBaPP Estimator)

UBaPP estimator is the solution to following convex program:

$$\min_{\delta_{p,\varepsilon} \in \mathcal{P}(\mathcal{Y},\Theta)} \int_{\Theta}\int_{\mathcal{Y}}\int_{\Theta} L(\theta,\tilde{\theta})\delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y})q_{\theta}(\mathbf{y})\pi(\theta)d\theta d\mathbf{y}d\tilde{\theta} \quad \text{Risk}$$

$$\text{s.t.} \quad \delta_{p,\varepsilon}(\tilde{\theta} \mid S(\mathbf{x})) \leq e^{\varepsilon}\delta_{p,\varepsilon}\left(\tilde{\theta} \mid S\left(\mathbf{x}'\right)\right), \text{ for each } \tilde{\theta} \in \Theta$$

$$\text{and } \mathbf{x}, \mathbf{x}' \in \mathcal{X} \text{ s.t. } d\left(\mathbf{x}, \mathbf{x}'\right) = 1$$

$$\int_{\Theta} \delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y})d\mathbf{y} = 1, \text{ for each } \mathbf{y} \in \mathcal{Y}$$

$$\delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) \geq 0, \text{ for each } \mathbf{y} \in \mathcal{Y}, \tilde{\theta} \in \Theta$$

Unified Approach (UBaPP Estimator)

UBaPP estimator is the solution to following convex program:

$$\min_{\delta_{p,\varepsilon} \in \mathcal{P}(\mathcal{Y},\Theta)} \int_{\Theta} \int_{\mathcal{Y}} \int_{\Theta} L(\theta, \tilde{\theta}) \delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) q_{\theta}(\mathbf{y}) \pi(\theta) d\theta d\mathbf{y} d\tilde{\theta}$$

s.t.

DP constraint

$$\delta_{p,\varepsilon}(\tilde{\theta} \mid S(\mathbf{x})) \leq e^{\varepsilon} \delta_{p,\varepsilon}\left(\tilde{\theta} \mid S(\mathbf{x}')\right), \text{ for each } \tilde{\theta} \in \Theta$$

$$\text{and } \mathbf{x}, \mathbf{x}' \in \mathcal{X} \text{ s.t. } d(\mathbf{x}, \mathbf{x}') = 1$$

$$\int_{\Theta} \delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) d\mathbf{y} = 1, \text{ for each } \mathbf{y} \in \mathcal{Y}$$

$$\delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) \geq 0, \text{ for each } \mathbf{y} \in \mathcal{Y}, \tilde{\theta} \in \Theta$$

Unified Approach (UBaPP Estimator)

UBaPP estimator is the solution to following convex program:

$$\min_{\delta_{p,\varepsilon} \in \mathcal{P}(\mathcal{Y},\Theta)} \int_{\Theta} \int_{\mathcal{Y}} \int_{\Theta} L(\theta, \tilde{\theta}) \delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) q_{\theta}(\mathbf{y}) \pi(\theta) d\theta d\mathbf{y} d\tilde{\theta}$$

$$\text{s.t.} \quad \delta_{p,\varepsilon}(\tilde{\theta} \mid S(\mathbf{x})) \leq e^{\varepsilon} \delta_{p,\varepsilon}\left(\tilde{\theta} \mid S\left(\mathbf{x}'\right)\right), \text{ for each } \tilde{\theta} \in \Theta$$

$$\text{and } \mathbf{x}, \mathbf{x}' \in \mathcal{X} \text{ s.t. } d\left(\mathbf{x}, \mathbf{x}'\right) = 1$$

$$\int_{\Theta} \delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) d\mathbf{y} = 1, \text{ for each } \mathbf{y} \in \mathcal{Y}$$

$$\delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) \geq 0, \text{ for each } \mathbf{y} \in \mathcal{Y}, \tilde{\theta} \in \Theta$$

Randomization constraint

Unified Approach (UBaPP Estimator)

UBaPP estimator is the solution to following convex program:

$$\min_{\delta_{p,\varepsilon} \in \mathcal{P}(\mathcal{Y},\Theta)} \int_{\Theta} \int_{\mathcal{Y}} \int_{\Theta} L(\theta, \tilde{\theta}) \delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) q_{\theta}(\mathbf{y}) \pi(\theta) d\theta d\mathbf{y} d\tilde{\theta}$$

$$\text{s.t.} \quad \delta_{p,\varepsilon}(\tilde{\theta} \mid S(\mathbf{x})) \leq e^{\varepsilon} \delta_{p,\varepsilon}\left(\tilde{\theta} \mid S\left(\mathbf{x}'\right)\right), \text{ for each } \tilde{\theta} \in \Theta$$

$$\text{and } \mathbf{x}, \mathbf{x}' \in \mathcal{X} \text{ s.t. } d\left(\mathbf{x}, \mathbf{x}'\right) = 1$$

$$\int_{\Theta} \delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) d\mathbf{y} = 1, \text{ for each } \mathbf{y} \in \mathcal{Y}$$

$$\delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) \geq 0, \text{ for each } \mathbf{y} \in \mathcal{Y}, \tilde{\theta} \in \Theta$$

UBaPP is optimal by construction!

Unified Approach (UBaPP Estimator)

UBaPP estimator is the solution to following convex program:

$$\min_{\delta_{p,\varepsilon} \in \mathcal{P}(\mathcal{Y},\Theta)} \int_{\Theta} \int_{\mathcal{Y}} \int_{\Theta} ||\theta - \tilde{\theta}||^2 \delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) q_{\theta}(\mathbf{y}) \pi(\theta) d\theta d\mathbf{y} d\tilde{\theta}$$

$$\text{s.t.} \quad \delta_{p,\varepsilon}(\tilde{\theta} \mid S(\mathbf{x})) \leq e^{\varepsilon} \delta_{p,\varepsilon}\left(\tilde{\theta} \mid S\left(\mathbf{x}'\right)\right), \text{ for each } \tilde{\theta} \in \Theta$$

$$\text{and } \mathbf{x}, \mathbf{x}' \in \mathcal{X} \text{ s.t. } d\left(\mathbf{x}, \mathbf{x}'\right) = 1$$

$$\int_{\Theta} \delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) d\mathbf{y} = 1, \text{ for each } \mathbf{y} \in \mathcal{Y}$$

$$\delta_{p,\varepsilon}(\tilde{\theta} \mid \mathbf{y}) \geq 0, \text{ for each } \mathbf{y} \in \mathcal{Y}, \tilde{\theta} \in \Theta$$

UBaPP is optimal by construction!

Outline

Unified Approach (UBaPP Estimator)

UBaPP Estimator for Finite Case

Numerical Example

Conclusion

UBaPP Estimator for Finite Case

UBaPP estimator $\equiv$ solution to a linear program:

$$\begin{aligned}
\min_{\mathbf{P} \in \mathbb{R}^{|\Theta| \times |\mathcal{Y}|}} \quad & \mathrm{tr}(\mathbf{Q}\, \mathrm{diag}(\boldsymbol{\pi}) \mathbf{L}\mathbf{P}) \\
\text{s.t.} \quad & \mathbf{P}_{k,i} \leq e^{\varepsilon} \mathbf{P}_{k,i'}, \text{ for all } k \in \{1, \ldots, |\Theta|\} \\
& \text{and } i, i' \in \{1, \ldots, |\mathcal{Y}|\} \text{ s.t. } d(\mathbf{x}_i, \mathbf{x}_{i'}) = 1 \\
& \mathbf{1}^T \mathbf{P} = \mathbf{1}^T \\
& \mathbf{P} \geq 0
\end{aligned}$$

UBaPP Estimator for Finite Case

UBaPP estimator $\equiv$ solution to a linear program:

$$\min_{\mathbf{P}\in\mathbb{R}^{|\Theta|\times|\mathcal{Y}|}} \operatorname{tr}(\mathbf{Q}\,\operatorname{diag}(\boldsymbol{\pi})\mathbf{L}\mathbf{P})$$

$$\text{s.t.} \quad \mathbf{P}_{k,i} \leq e^{\varepsilon}\mathbf{P}_{k,i'}, \text{ for all } k \in \{1,\ldots,|\Theta|\}$$

$$\text{and } i, i' \in \{1,\ldots,|\mathcal{Y}|\} \text{ s.t. } d\left(\mathbf{x}_i, \mathbf{x}_{i'}\right) = 1$$

$$\mathbf{1}^T\mathbf{P} = \mathbf{1}^T$$

$$\mathbf{P} \geq 0$$

Solved using CVXPY!

Outline

Unified Approach (UBaPP Estimator)

UBaPP Estimator for Finite Case

Numerical Example

Conclusion

Numerical Example

Private estimation of Bernoulli parameter (θ) using K trials

- $\Theta = [0, 1]$

Numerical Example

Private estimation of Bernoulli parameter (θ) using K trials

- $\Theta = [0, 1] \overset{\text{discretize}}{\Longrightarrow} \theta_j \overset{i.i.d.}{\sim} \pi,\ j \in \{1, \ldots, M\}$

Numerical Example

Private estimation of Bernoulli parameter (θ) using K trials

- $\Theta = [0, 1] \overset{\text{discretize}}{\Longrightarrow} \theta_j \overset{i.i.d.}{\sim} \pi, \, j \in \{1, \ldots, M\}$
- $\mathcal{Y} = \{0, \ldots, K\}$

Numerical Example

Private estimation of Bernoulli parameter (θ) using K trials

- $\Theta = [0, 1] \overset{\text{discretize}}{\Longrightarrow} \theta_j \overset{i.i.d.}{\sim} \pi,\ j \in \{1, \ldots, M\}$
- $\mathcal{Y} = \{0, \ldots, K\}$
- $S(\boldsymbol{x}) = \sum_{i=1}^{K} x_i \quad x_i \overset{i.i.d.}{\sim} \text{Ber}(\theta_j),\ i = 1, \ldots, K$

Numerical Example

Private estimation of Bernoulli parameter (θ) using K trials

- $\Theta = [0, 1] \overset{\text{discretize}}{\Longrightarrow} \theta_j \overset{i.i.d.}{\sim} \pi, \; j \in \{1, \dots, M\}$
- $\mathcal{Y} = \{0, \dots, K\}$
- $S(\boldsymbol{x}) = \sum_{i=1}^{K} x_i \quad x_i \overset{i.i.d.}{\sim} \text{Ber}(\theta_j), \; i = 1, \dots, K$

Laplace Bayes Private Point (LBaPP) estimator for this setup:

$$\delta_{lpb,\varepsilon} = \frac{1}{K+2}\left(\sum_{i=1}^{K} x_i + 1\right) + \text{Lap}\left(0, \frac{1}{(K+2)\varepsilon}\right)$$

Numerical Example

Private estimation of Bernoulli parameter (θ) using K trials

- $\Theta = [0, 1] \overset{\text{discretize}}{\Longrightarrow} \theta_j \overset{i.i.d.}{\sim} \pi,\ j \in \{1, \ldots, M\}$
- $\mathcal{Y} = \{0, \ldots, K\}$
- $S(\boldsymbol{x}) = \sum_{i=1}^{K} x_i \quad x_i \overset{i.i.d.}{\sim} \text{Ber}(\theta_j),\ i = 1, \ldots, K$

Laplace Bayes Private Point (LBaPP) estimator for this setup:

$$\delta_{lpb,\varepsilon} = \underbrace{\frac{1}{K+2}\left(\sum_{i=1}^{K} x_i + 1\right)}_{\text{non-private estimate}} + \text{Lap}\left(0, \frac{1}{(K+2)\varepsilon}\right)$$

Numerical Example

Private estimation of Bernoulli parameter (θ) using K trials

- $\Theta = [0, 1] \overset{\text{discretize}}{\Longrightarrow} \theta_j \overset{i.i.d.}{\sim} \pi,\ j \in \{1, \ldots, M\}$
- $\mathcal{Y} = \{0, \ldots, K\}$
- $S(\boldsymbol{x}) = \sum_{i=1}^{K} x_i \quad x_i \overset{i.i.d.}{\sim} \text{Ber}(\theta_j),\ i = 1, \ldots, K$

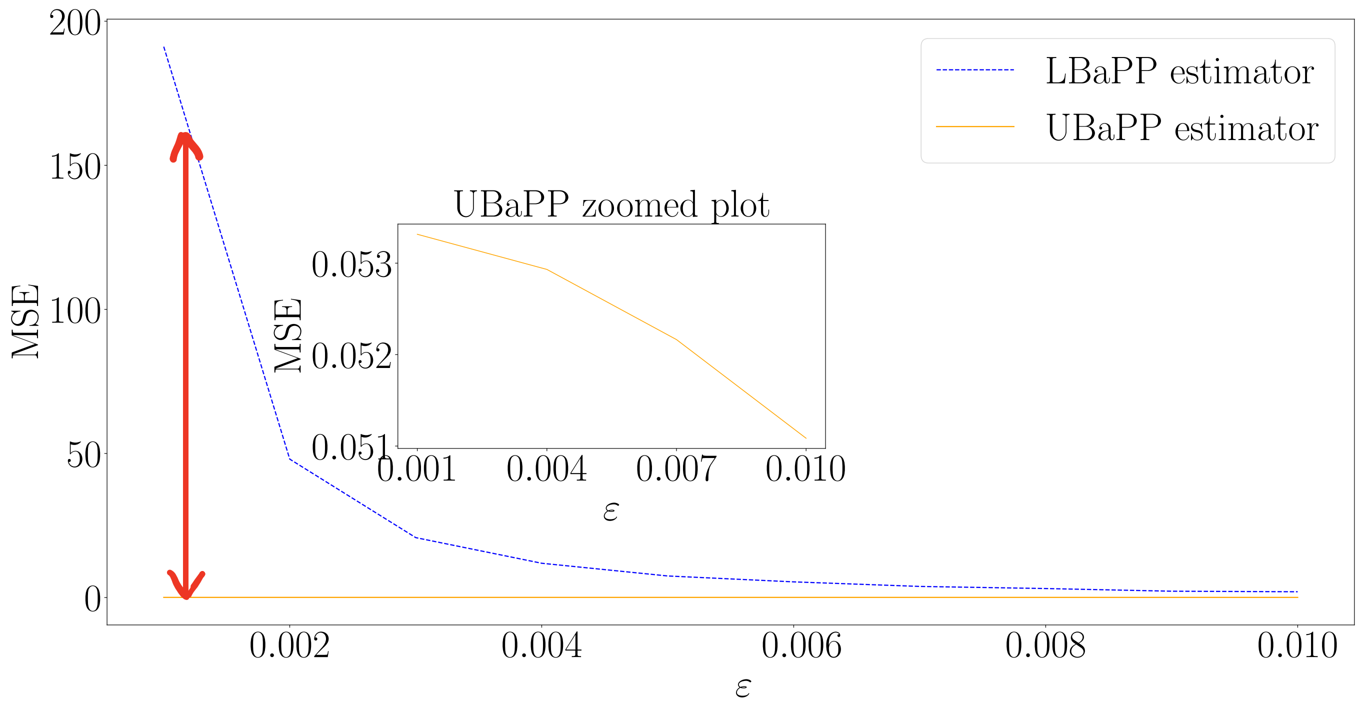
Laplace Bayes Private Point (LBaPP) estimator for this setup:

$$\delta_{lpb,\varepsilon} = \underbrace{\frac{1}{K+2}\left(\sum_{i=1}^{K} x_i + 1\right)}_{\text{non-private estimate}} + \underbrace{\text{Lap}\left(0, \frac{1}{(K+2)\varepsilon}\right)}_{\text{Laplace noise}}$$

Plots (MSE v.s. ε)

For a fixed K ($K = 100$)

High privacy regime

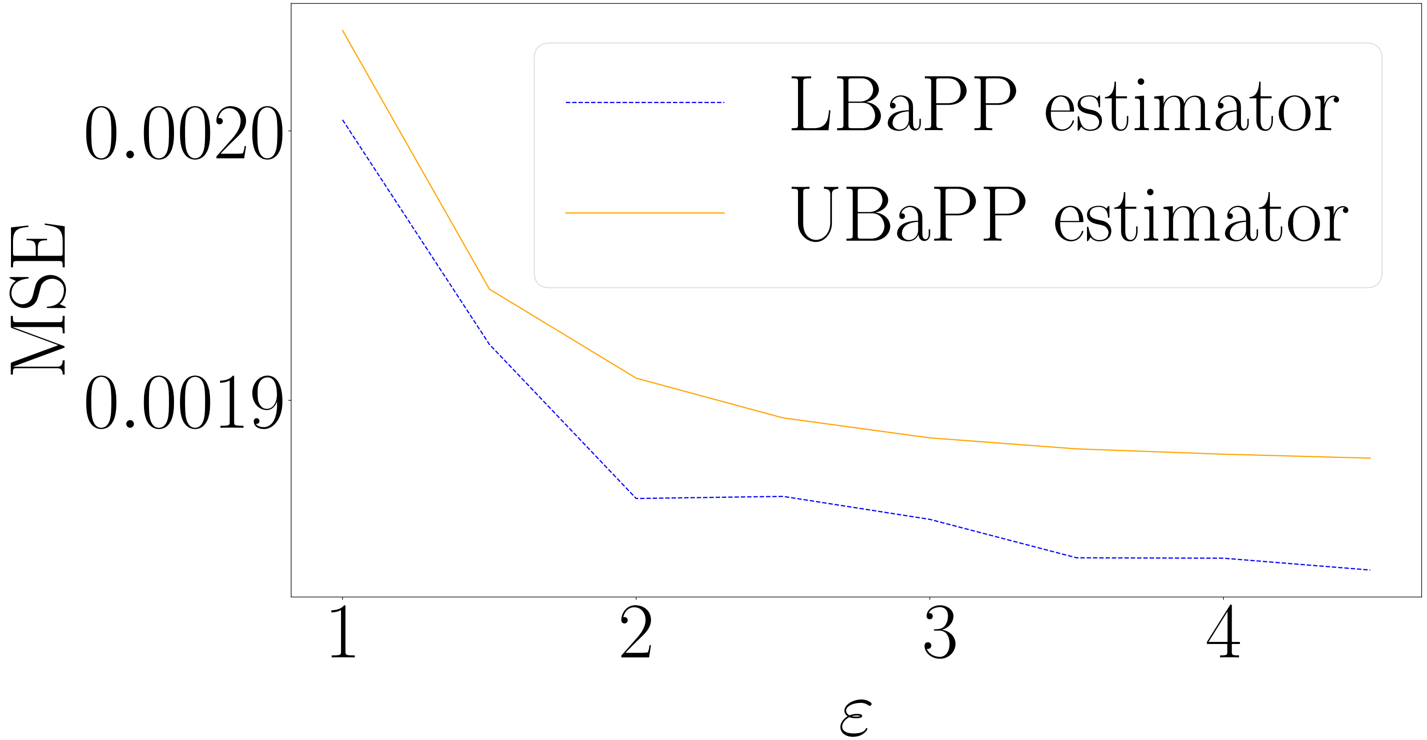


High accuracy is achieved by our approach!

Plots (MSE v.s. ε)

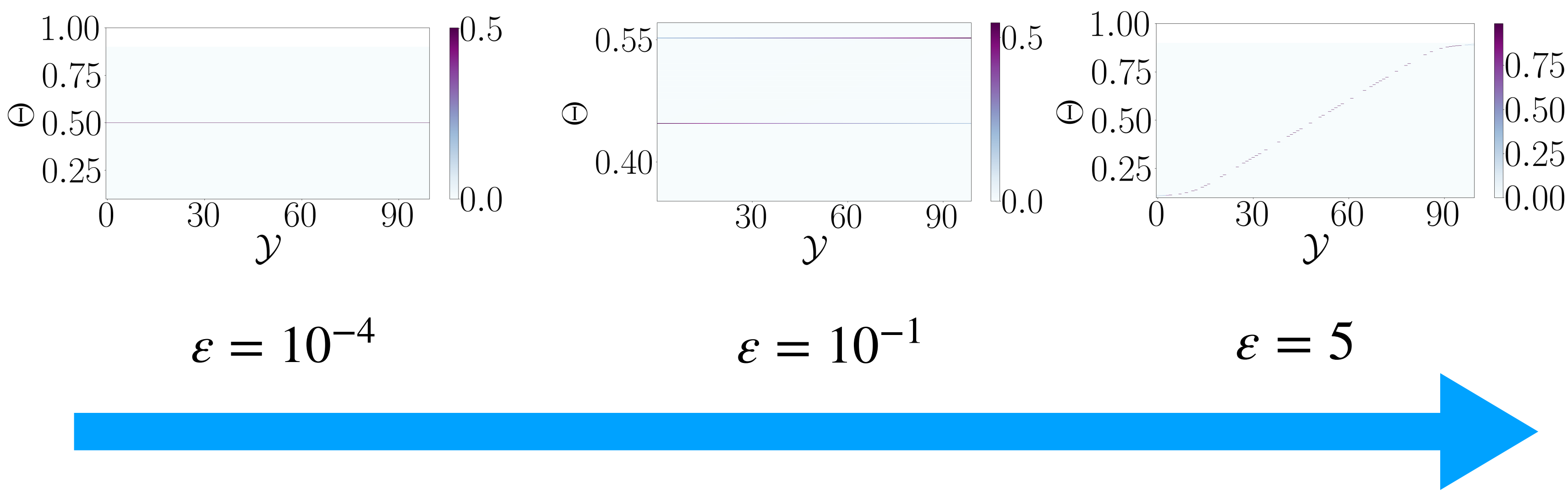
For a fixed K ($K = 100$)

Low privacy regime



Comparable performance!

Heat Maps



- $\varepsilon = 10^{-4}$: Deterministic estimate, independent of $\boldsymbol{y}$, no inference about $\boldsymbol{x}$
- $\varepsilon = 10^{-1}$: Randomized estimate, still independent of $\boldsymbol{y}$, still no inference about $\boldsymbol{x}$
- $\varepsilon = 5$: Deterministic estimate, strongly dependent on $\boldsymbol{y}$, complete inference about $\boldsymbol{x}$

Outline

Unified Approach (UBaPP Estimator)

UBaPP Estimator for Finite Case

Numerical Example

Conclusion

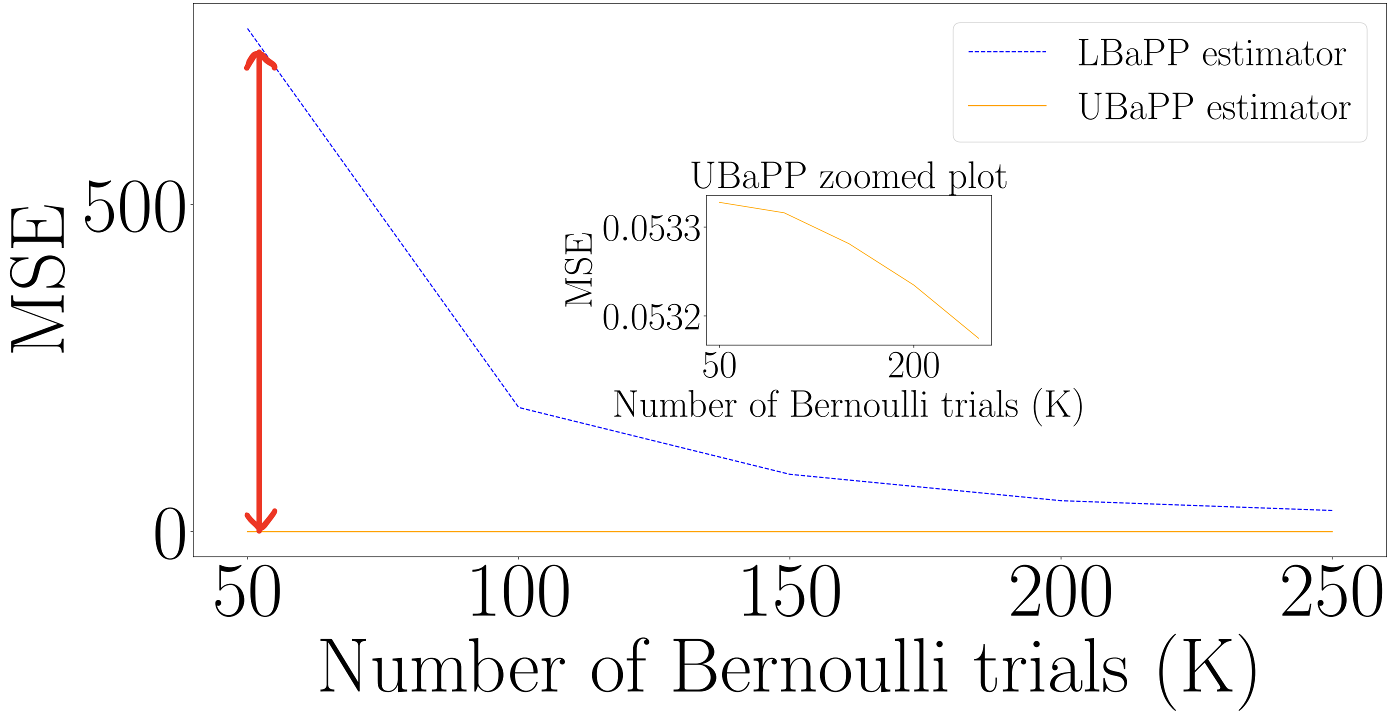
Conclusion

- Provided a unified approach to yield Bayes point estimate subject to differential privacy
- The "noise" is implicitly "added" by randomizing the estimator directly
- Demonstrated promising result in the limiting case (high-privacy regime) for the finite case via a numerical example
- Future work: Analyze the UBaPP estimator for high dimensional parameter and observation space

Thank You

Plots (MSE v.s. K)

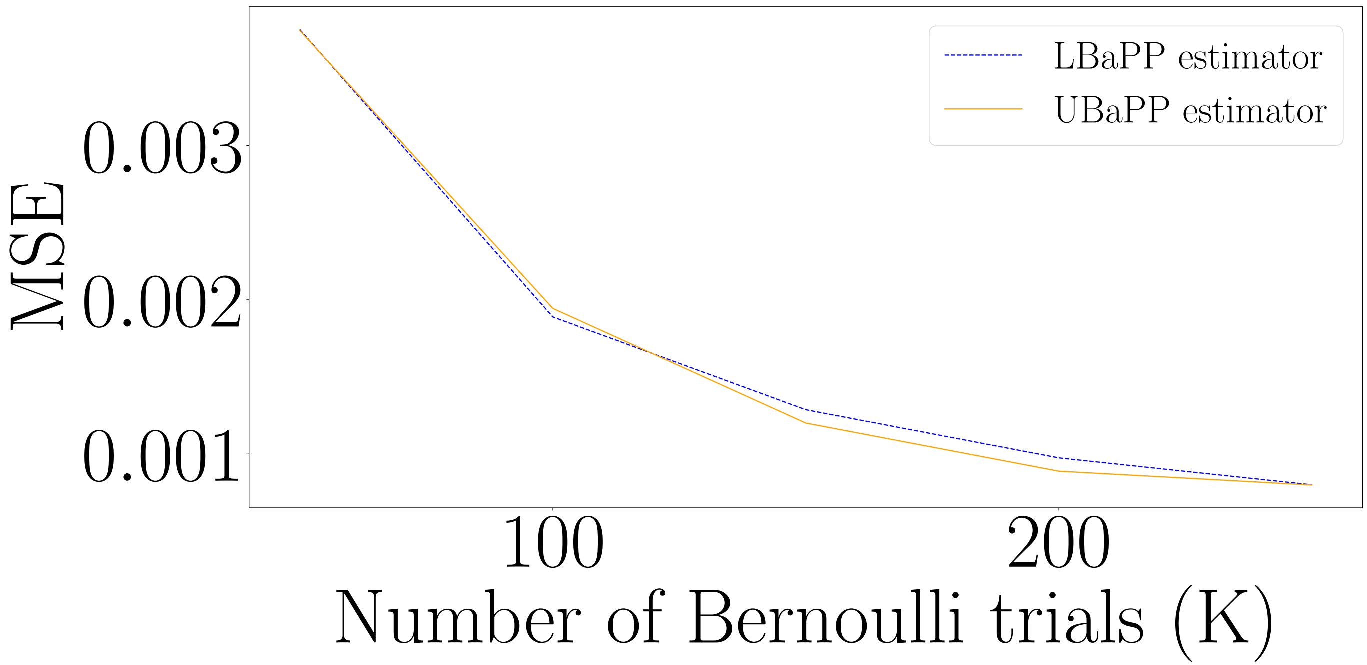
High privacy regime ($\varepsilon = 10^{-3}$)



High gain in sample complexity!

Plots (MSE v.s. K)

Low privacy regime ($\varepsilon = 5$)



Comparable sample complexity!