From Data to Control: A Two-Stage Simulation-Based Approach




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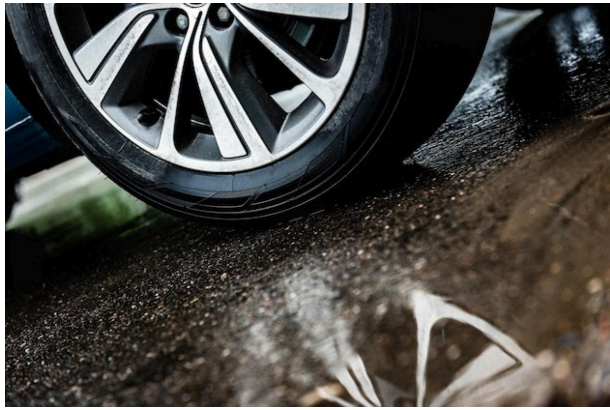
Motivation

- **Control systems** often required to **operate in uncertain/varying** conditions;

- **Some plant knowledge** is usually **available**. But **key parameters might change** over time;

- This **undermines to some extent traditional model-based approaches**, requiring re-tuning of the controller.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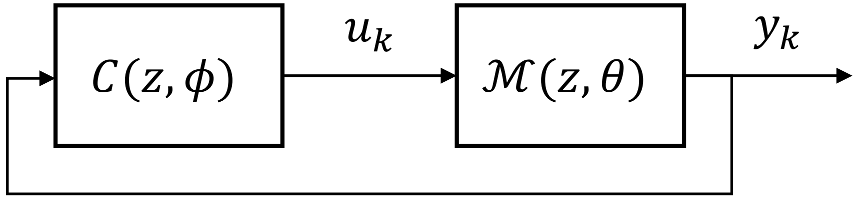

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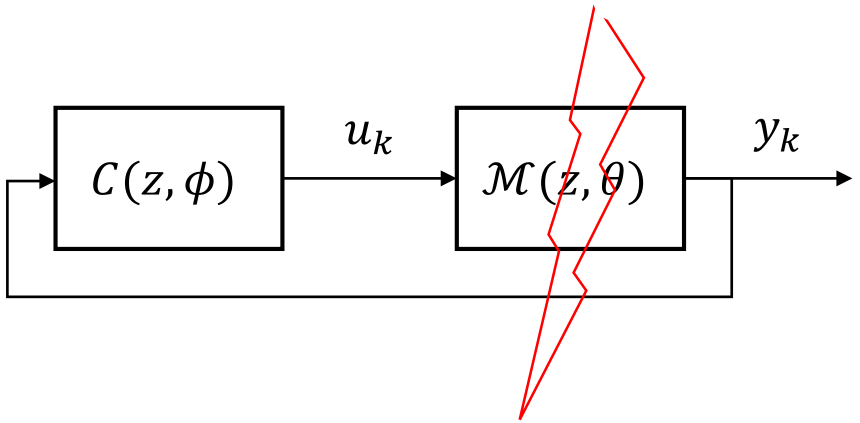
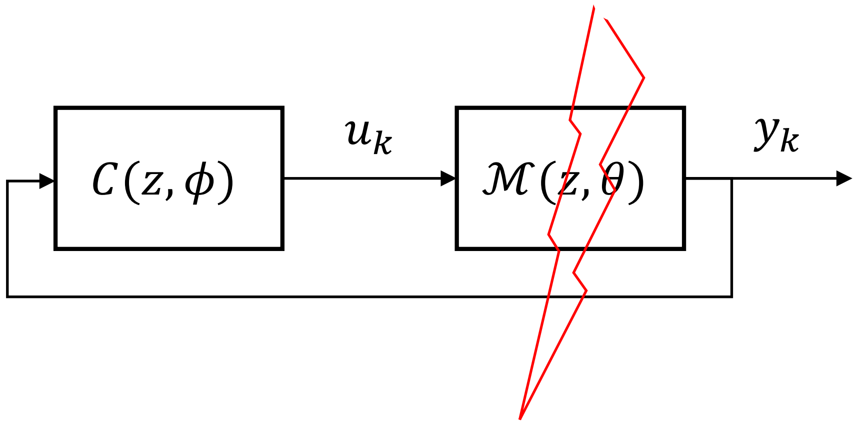


$$\theta_\delta \in \Theta = \{\theta : ||\theta - \theta_0||_2 \leq \rho\}$$

perturbed parameters

maximum deviation (ρ)

Motivation



$$\theta_\delta \in \Theta = \{\theta : ||\theta - \theta_0||_2 \leq \rho\}$$

perturbed parameters

maximum deviation (ρ) known

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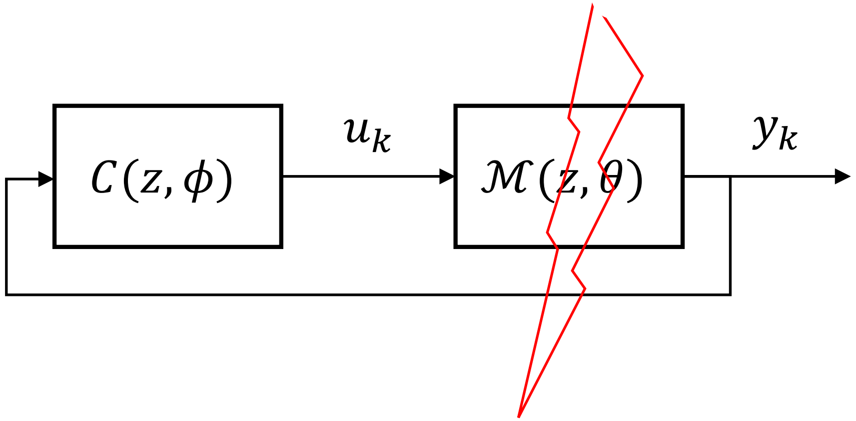


$$\theta_\delta \in \Theta = \{\theta : ||\theta - \theta_0||_2 \leq \rho\}$$

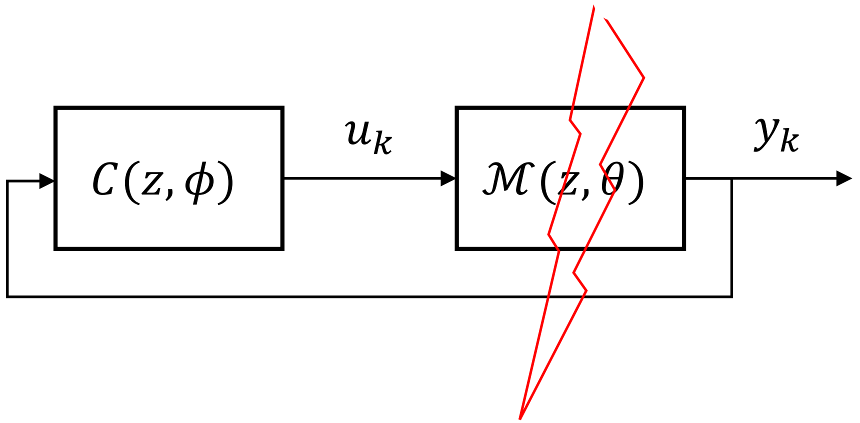
perturbed parameters

maximum deviation (ρ) known

$\mathcal{M}(z,\theta)$
(simulator)

available

Motivation



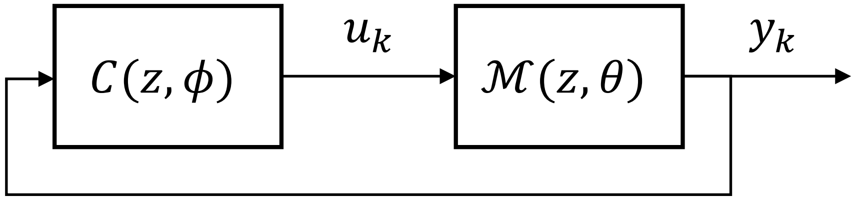
$$\theta_\delta \in \Theta = \{\theta : ||\theta - \theta_0||_2 \leq \rho\}$$

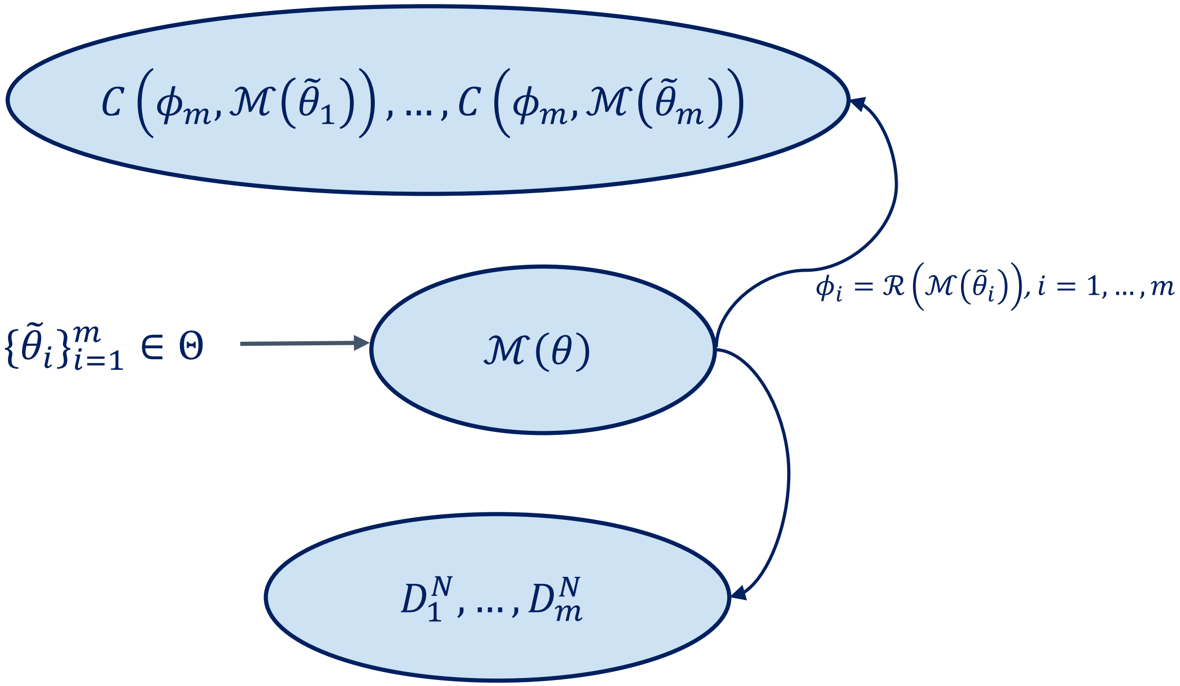
perturbed parameters

Can we leverage this information to fully automate controller tuning ?

- maximum deviation (ρ) — known
- $\mathcal{M}(z,\theta)$ (simulator) — available

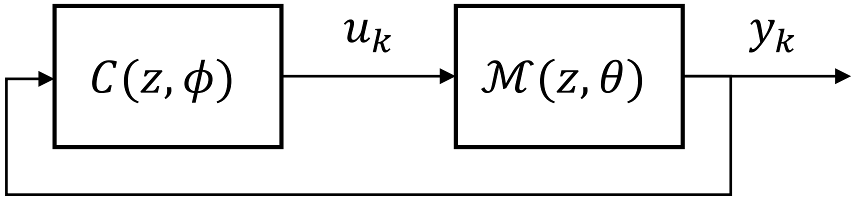
Setup



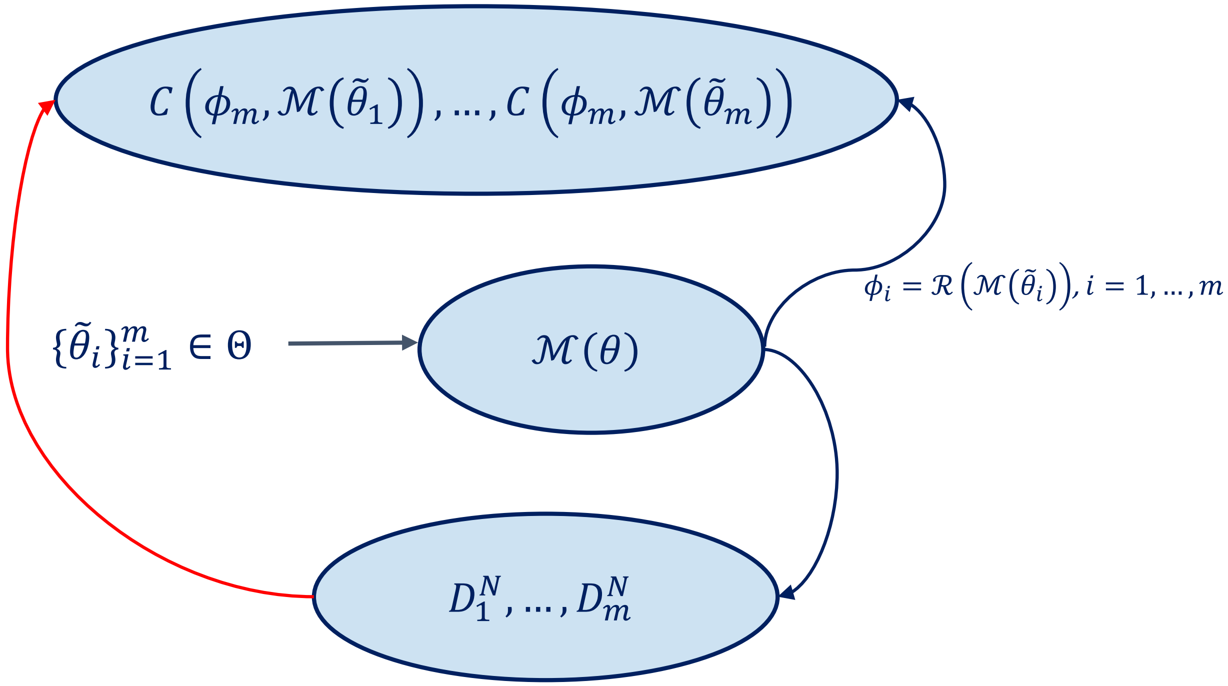


$\tilde{\theta}_1$	$D_1^N = \{(u_1^{(1)}, y_1^{(1)}), \dots, (u_N^{(1)}, y_N^{(1)})\}$	$C\left(\phi_1, \mathcal{M}(\tilde{\theta}_1)\right)$
$\tilde{\theta}_2$	$D_2^N = \{(u_1^{(2)}, y_1^{(2)}), \dots, (u_N^{(2)}, y_N^{(2)})\}$	$C\left(\phi_2, \mathcal{M}(\tilde{\theta}_2)\right)$
$\vdots$	$\vdots$	$\vdots$
$\tilde{\theta}_m$	$D_m^N = \{(u_1^{(m)}, y_1^{(m)}), \dots, (u_N^{(m)}, y_N^{(m)})\}$	$C\left(\phi_m, \mathcal{M}(\tilde{\theta}_m)\right)$

Setup



$$f^* = \underset{f}{\operatorname{argmin}} \frac{1}{m} \sum_{i=1}^{m} \left\| \phi_i - f\left(y_1^{(i)}, u_1^{(i)}, \dots, y_N^{(i)}, u_N^{(i)}\right) \right\|_2^2$$



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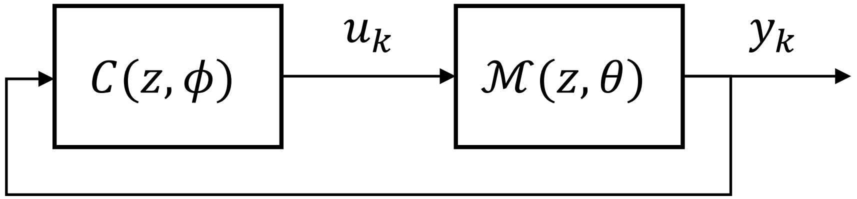
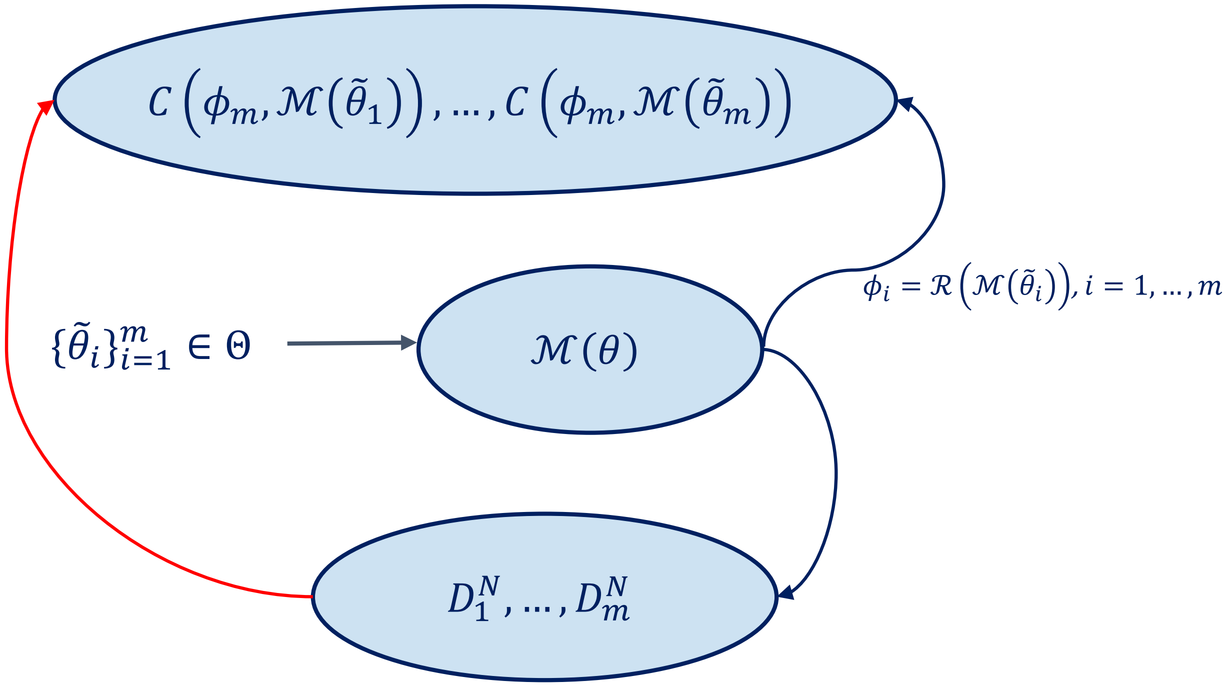


$$\boldsymbol{f}^* = \underset{\boldsymbol{f}}{\operatorname{argmin}} \frac{\mathbf{1}}{\boldsymbol{m}} \sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{m}} \left\| \boldsymbol{\phi}_{\boldsymbol{i}} - \boldsymbol{f}\left(\boldsymbol{y}_{\mathbf{1}}^{(\boldsymbol{i})}, \boldsymbol{u}_{\mathbf{1}}^{(\boldsymbol{i})}, \dots, \boldsymbol{y}_{\boldsymbol{N}}^{(\boldsymbol{i})}, \boldsymbol{u}_{\boldsymbol{N}}^{(\boldsymbol{i})}\right) \right\|_{\mathbf{2}}^{\mathbf{2}}$$

Use of Two-Stage parameter estimation paradigm to meta-learn $\boldsymbol{f}^*$



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Outline

- ❑Two-stage (TS) estimation paradigm
- ❑TS for controller tuning
- ❑Numerical study
- ❑Conclusion

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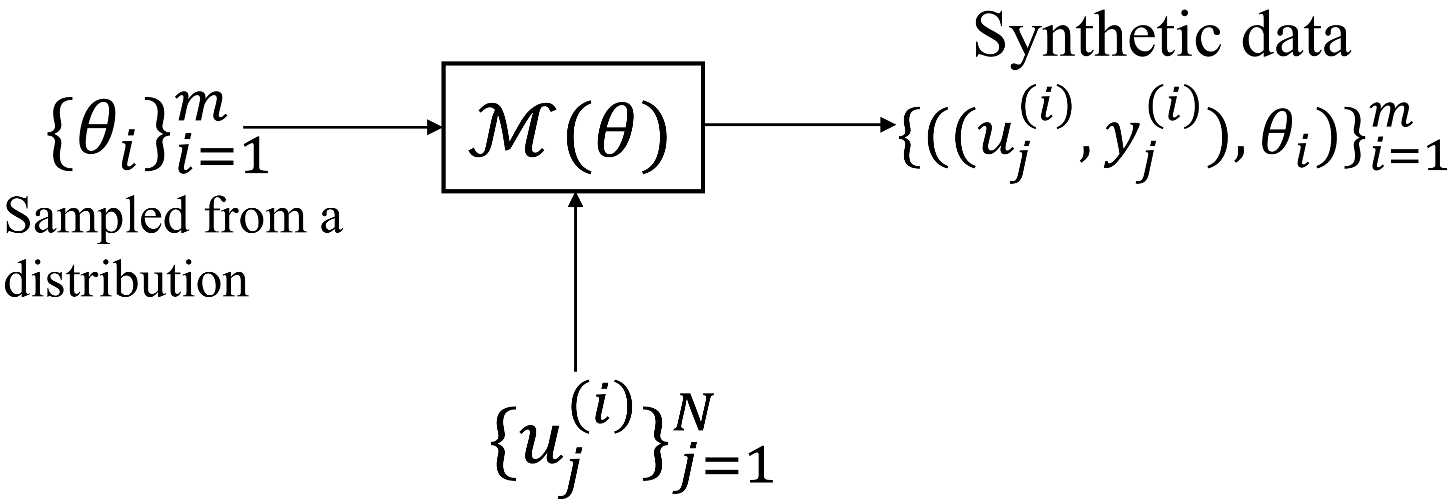
TS estimation paradigm



S. Garatti, S. Bittanti. "A new paradigm for parameter estimation in system modeling". Int. J. Adapt. Control Sig. Proc., 2013

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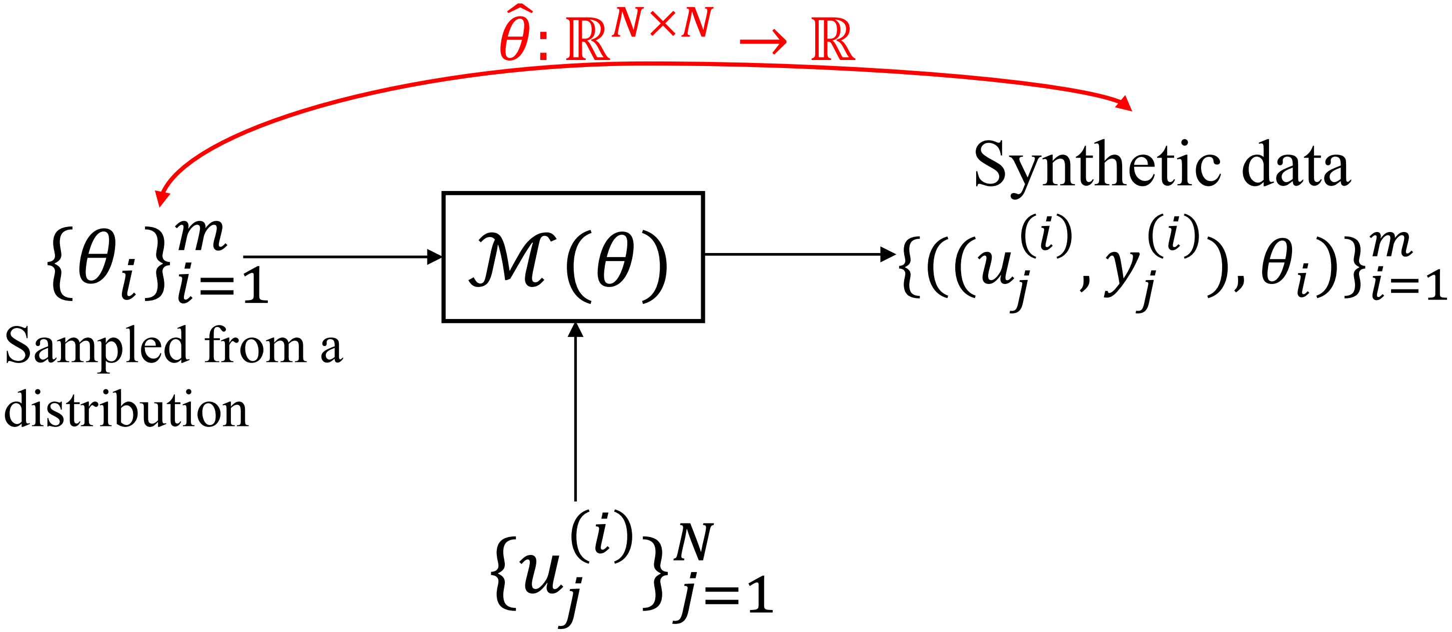

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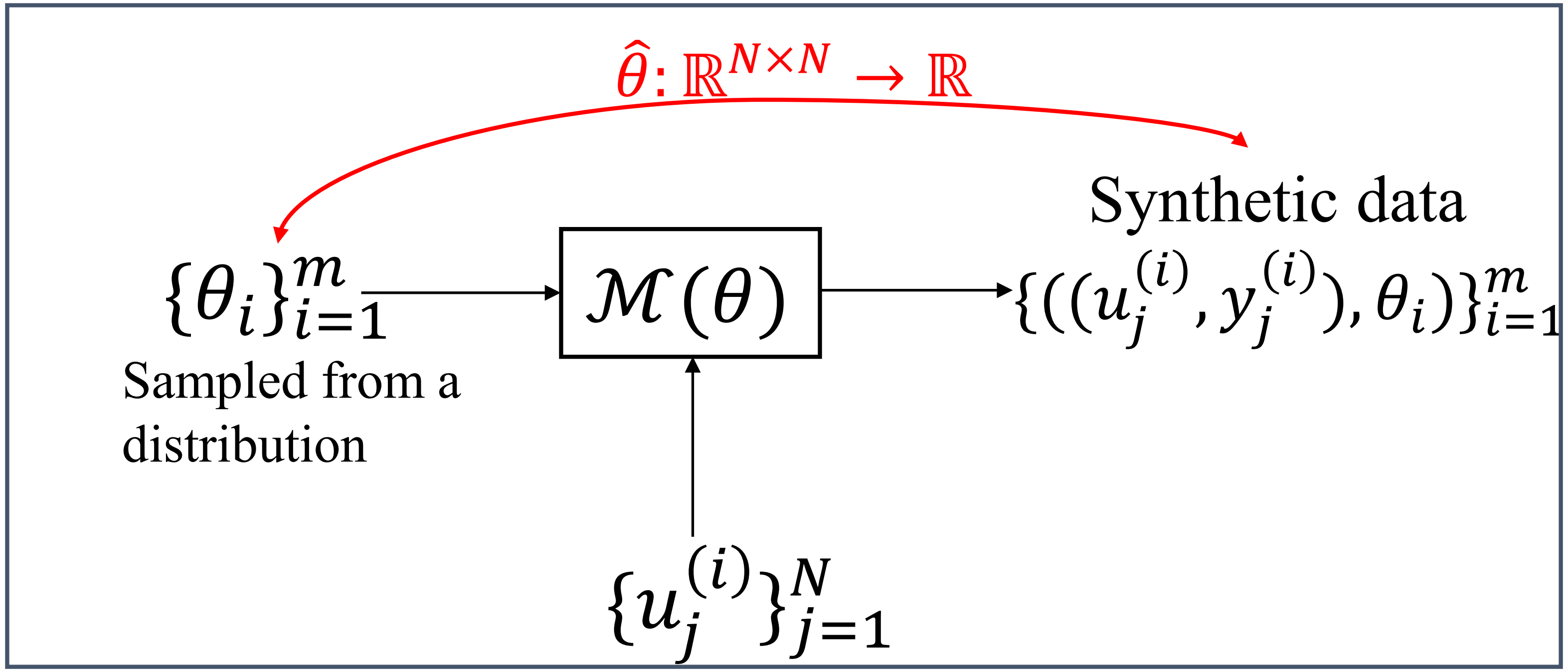
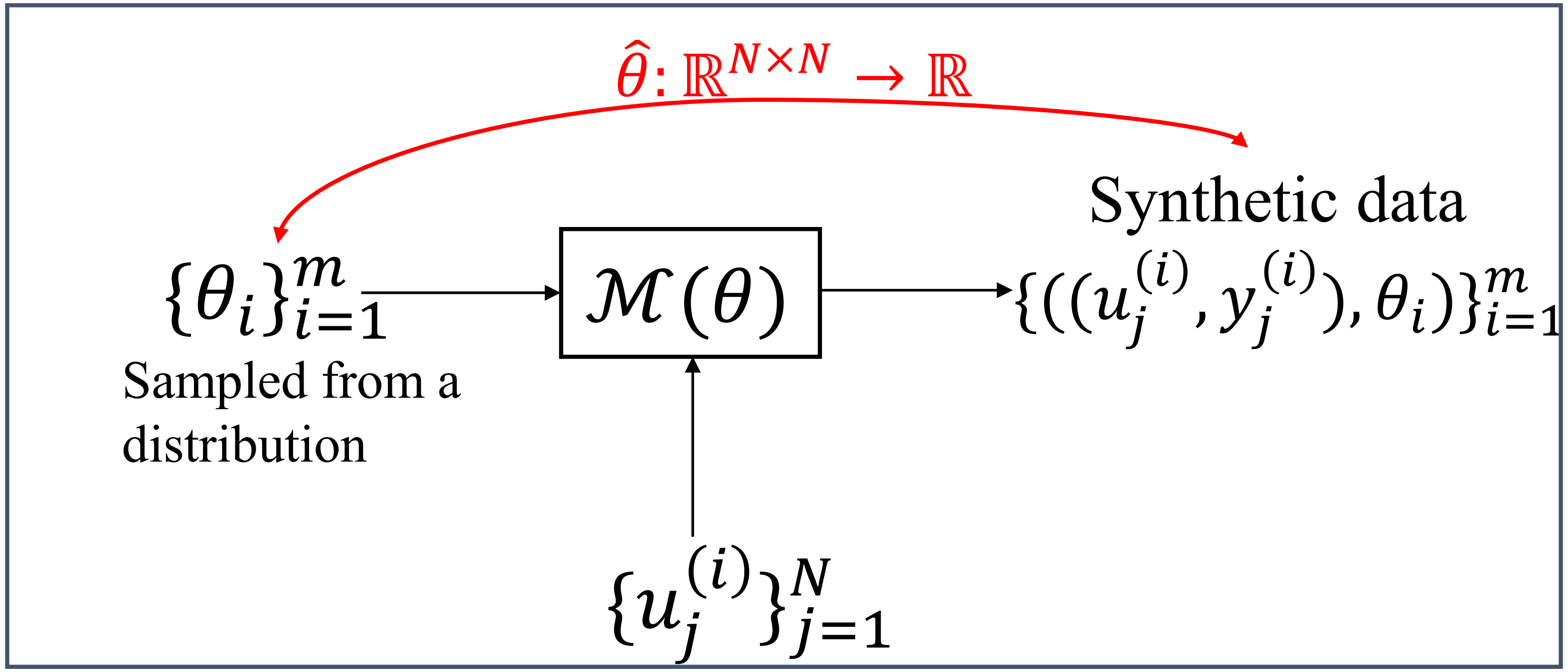


Inverse (meta) learning

TS estimation paradigm



Inverse (meta) learning

$$g^* = \arg\min_{g \in \mathcal{G}} \frac{1}{M} \sum_{i=1}^{m} \left\| \theta_i - g\left(h(y_1^{(i)}, u_1^{(i)}, \dots, y_N^{(i)}, u_N^{(i)}) \right) \right\|_2^2$$

$$\hat{\theta}_{TS} = g^* \circ h$$

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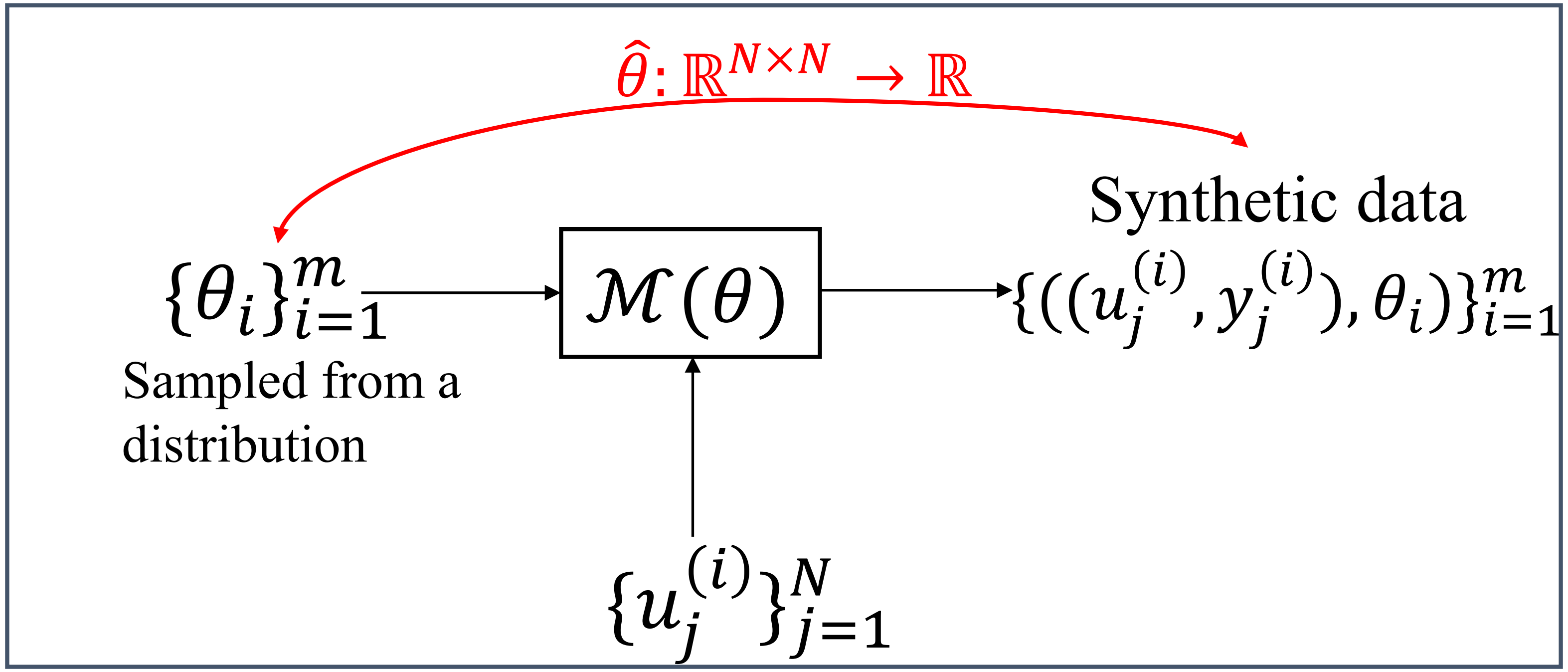
TS estimation paradigm



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$$\hat{\theta}_{TS} = g^* \circ h$$

$h: \mathbb{R}^{N\times N} \rightarrow \mathbb{R}^n$ Fixed
Data compressor

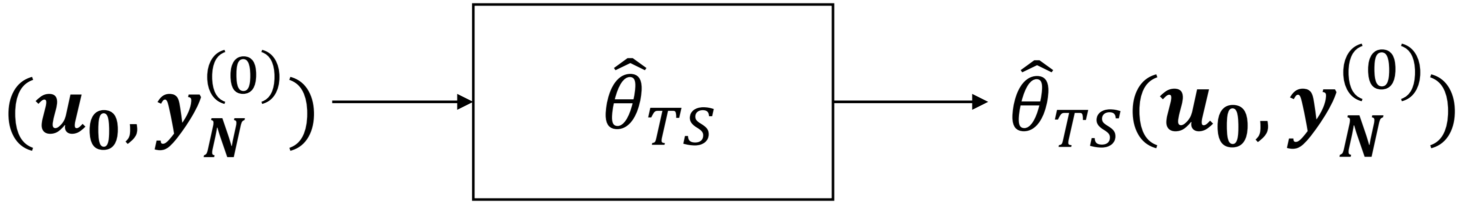
$g: \mathbb{R}^n \rightarrow \mathbb{R}^{d_\theta}$ Optimized
Function approximator

S. Garatti, S. Bittanti. "A new paradigm for parameter estimation in system modeling". Int. J. Adapt. Control Sig. Proc., 2013

How and why?

$\hat{\theta}_{TS}$ is an estimator functional

- Need to be tested on a new observation



Why promising?

- Compression step – helps in dimensionality reduction
- Second stage – A continuous map of compressed samples

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- ❑Two-stage (TS) estimation paradigm
- ❑TS for controller tuning
- ❑Numerical study
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TS for controller tuning

Objective: Learn function $\boldsymbol{f}^*$ such that:

$$f^* = \underset{f}{\operatorname{argmin}} \frac{1}{m} \sum_{i=1}^{m} \left\| \phi_i - f\left(y_1^{(i)}, u_1^{(i)}, \dots, y_N^{(i)}, u_N^{(i)}\right) \right\|_2^2 = g^* \circ h_N$$

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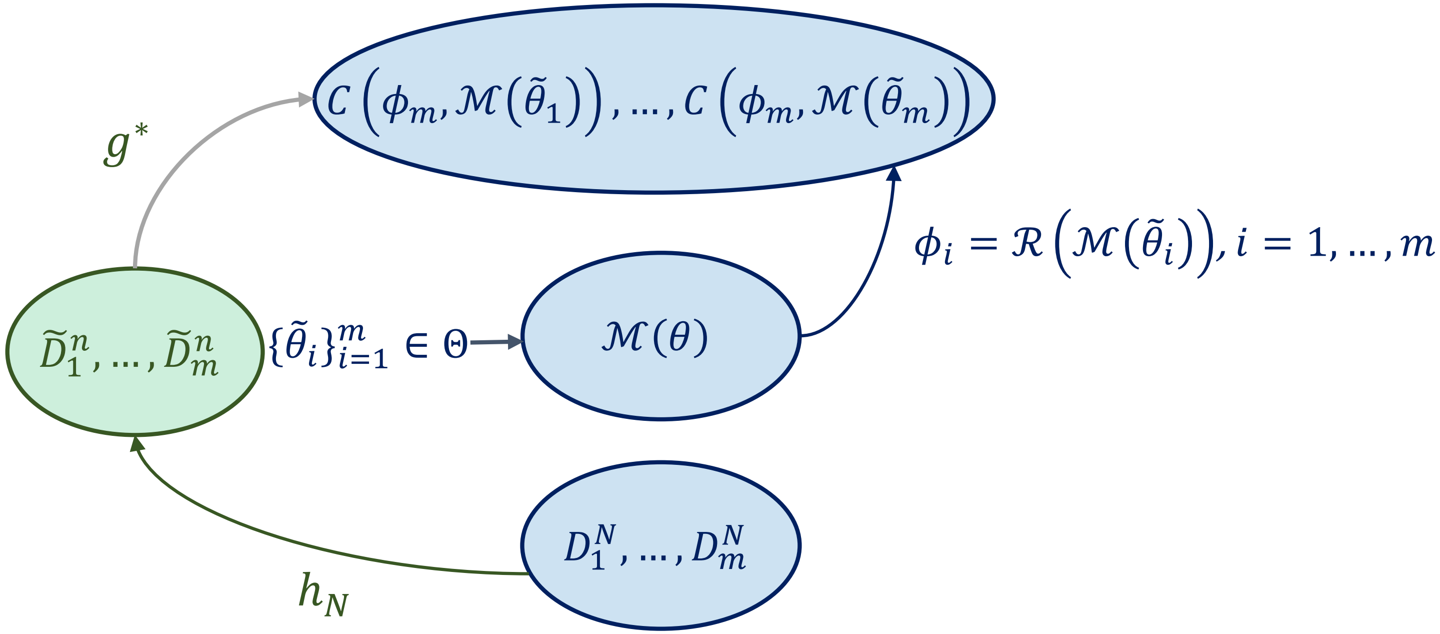

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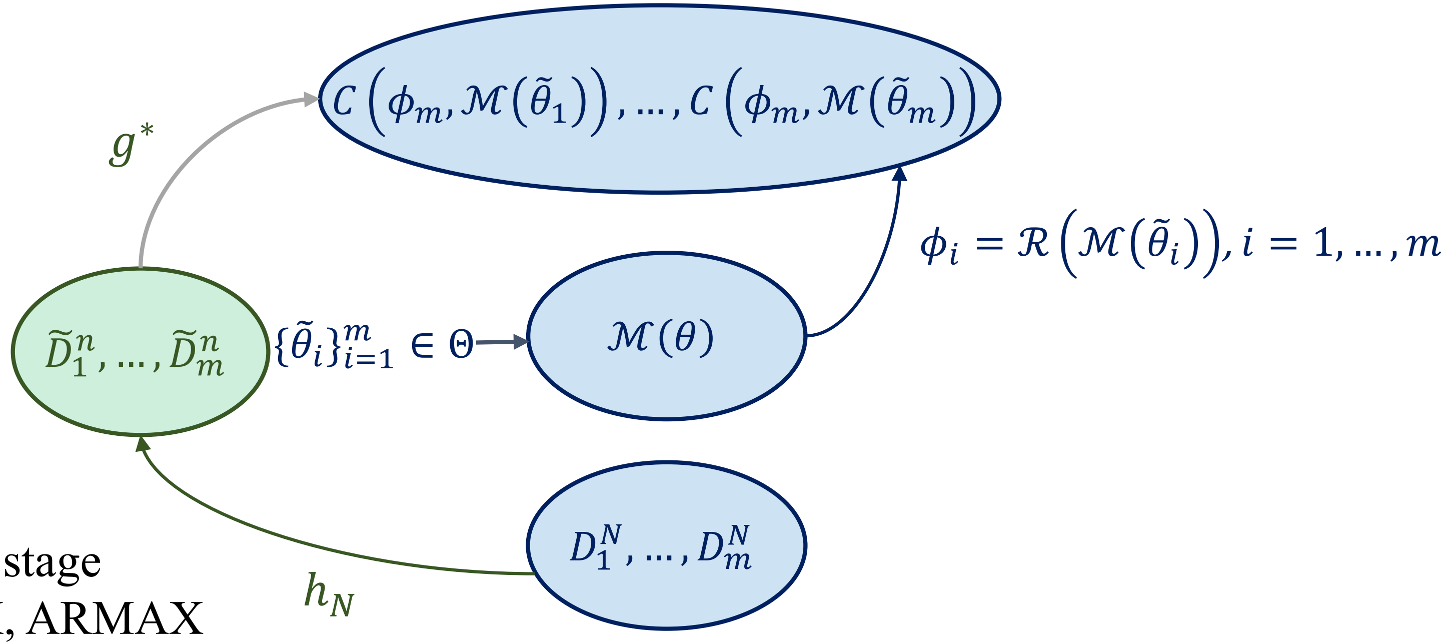

First stage
ARX, ARMAX

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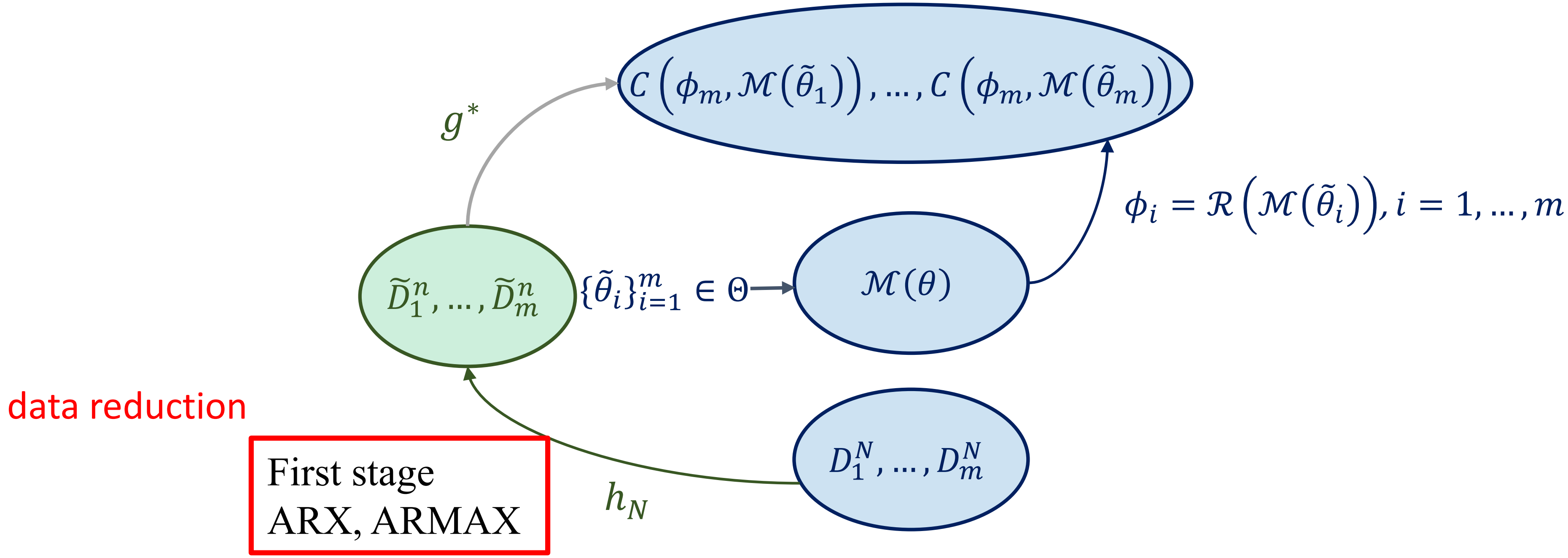

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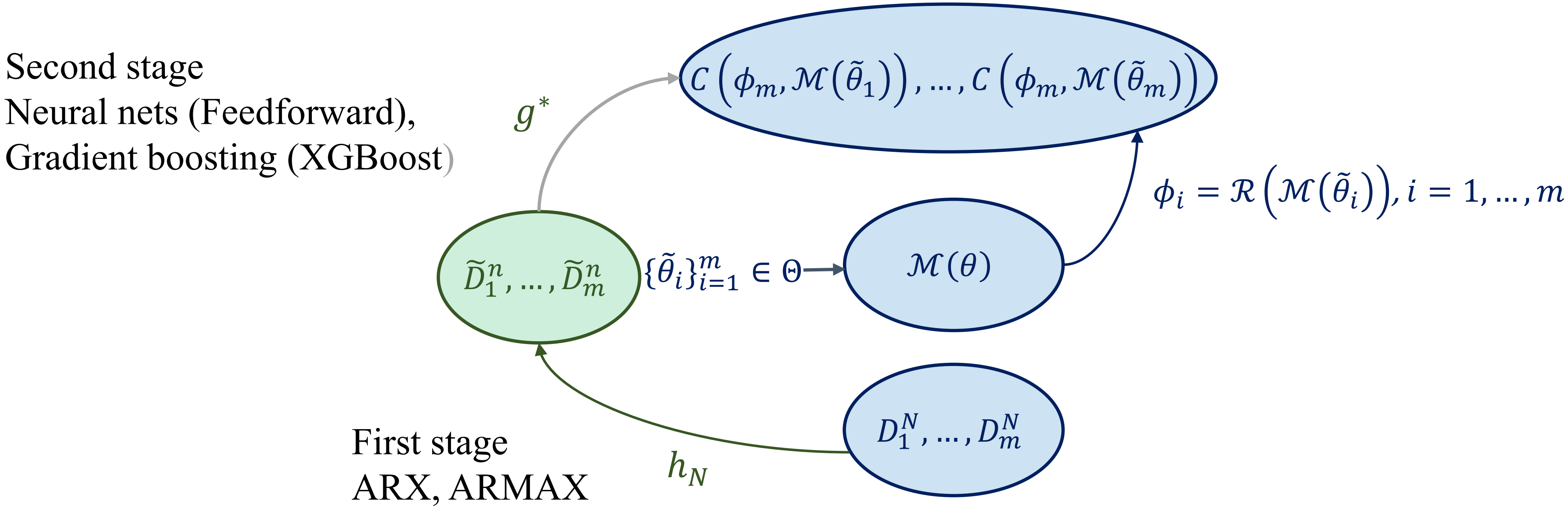

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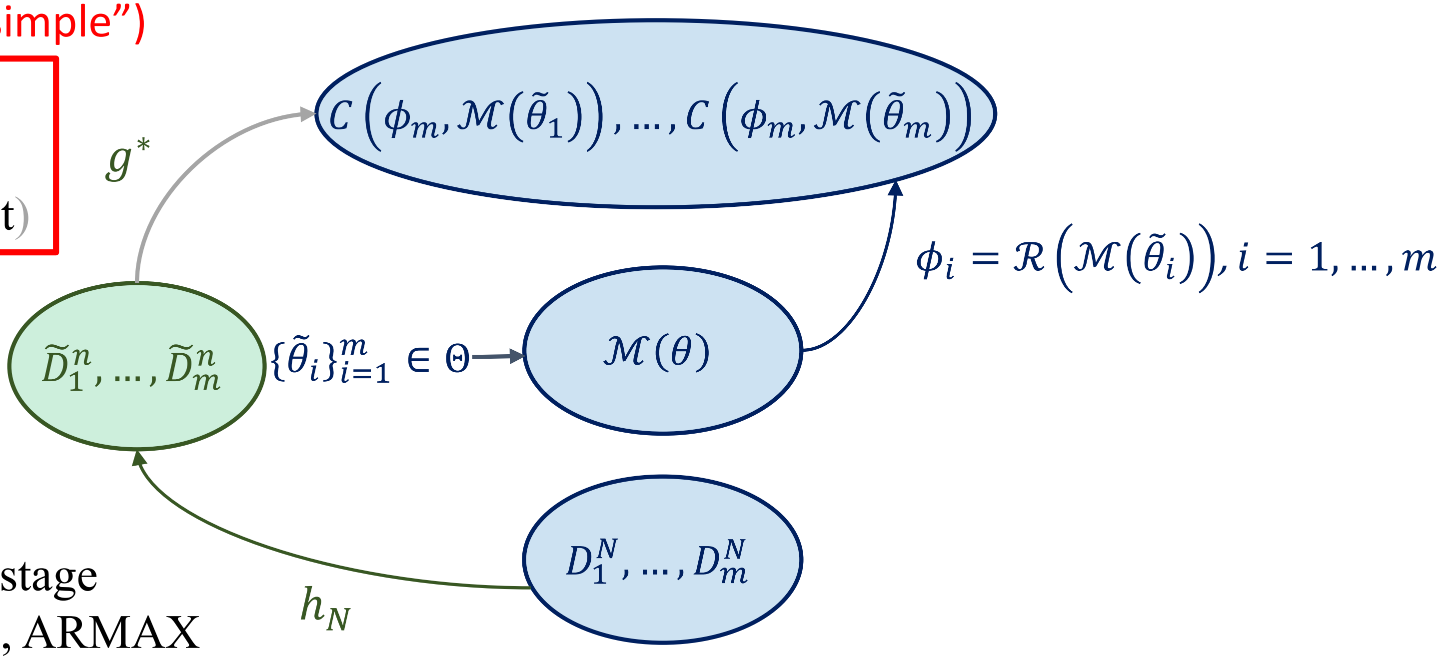
Function approximation ("simple")

Second stage
Neural nets (Feedforward),
Gradient boosting (XGBoost)



First stage
ARX, ARMAX

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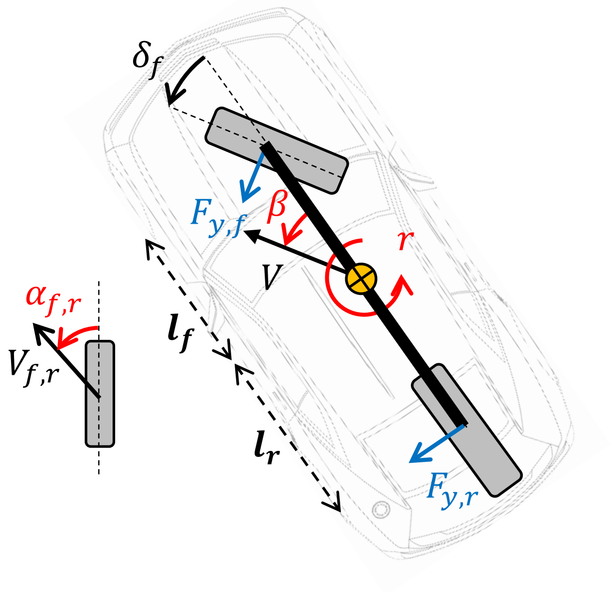
Numerical study

Objective: Achieve desired yaw-rate r



Vehicle dynamics:

state: $\boldsymbol{x} = (\beta, r, \alpha_f, \alpha_r, z, \delta_f)^T$
input $u = \delta_f^{cmd}$
output $y = r$

$$\dot{\beta} = -r - \frac{C_f \alpha_f}{M_{veh}\, v_x} - \frac{C_r \alpha_r}{M_{veh}\, v_x}$$

$$\dot{r} = -\frac{l_f C_f \alpha_f}{J_z} + \frac{l_r C_r \alpha_r}{J_z}$$

$$\dot{\alpha}_f = -\frac{v_x}{l_{rel,f}}(\alpha_f - \alpha_f^{kin}), \qquad \alpha_f^{kin} = -\delta_f + \beta + \frac{L_f}{v_x} r$$

$$\dot{\alpha}_r = -\frac{v_x}{l_{rel,r}}(\alpha_r - \alpha_r^{kin}), \qquad \alpha_r^{kin} = \beta - \frac{L_r}{v_x} r$$

$$\dot{z} = -\omega_n^2\, \delta_f + \omega_n^2 \delta_f^{cmd}$$

$$\dot{\delta}_f = z - 2\,\xi \omega_n \delta_f$$

M_{veh} [kg]	J_z [kgm^2]	l_f [m]	l_r [m]	C_f [N	C_r [N/rad]	T_s [s]	ω_n [rad/s]	ξ
1895	2400	1.18	1.53	$1.24 \cdot 10^5$	$1.66 \cdot 10^5$	0.01	2π . 5	0.9

1. T. Hiraoka et al, "Model-following sliding mode control for active four-wheel steering vehicle," Review of Automotive Engineering, 2004.

Numerical study

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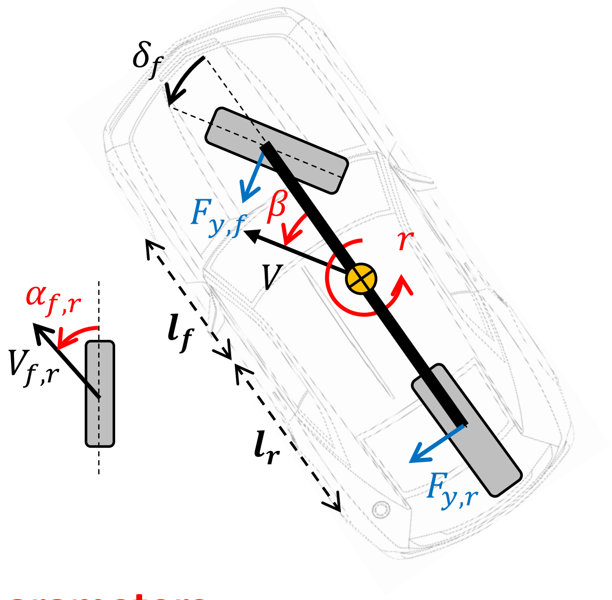

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Uncertain parameters

M_{veh} [kg]	J_z [kgm^2]	l_f [m]	l_r [m]	C_f [N	C_r [N/rad]	T_s [s]	ω_n [rad/s]	ξ
1895	2400	1.18	1.53	$1.24 \cdot 10^5$	$1.66 \cdot 10^5$	0.01	2π . 5	0.9

$$M_{veh} = M_0 + M_\delta$$
$$C_{f,r} = C_{f,r}^{nom}\, \mu_s$$
$$M_\delta \in [0,300]$$
$$\mu_s \in [0.3,1.1]$$

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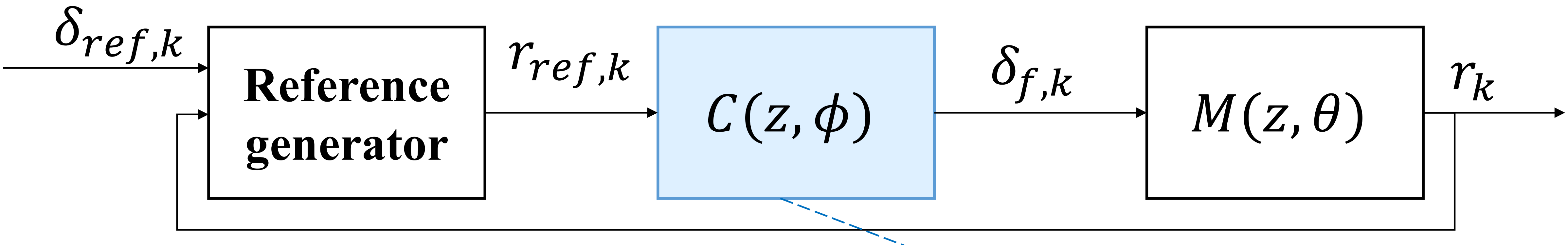
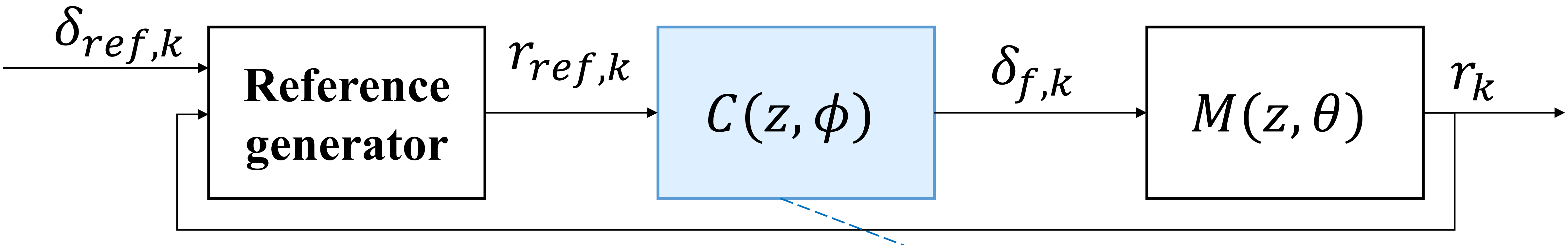
Control design



A **Proportional-Integral controller** suffices for our purposes. We design it via loop shaping, such as to guarantee phase margin $\phi_m \geq 60^{\circ}$ and cutting frequency $\omega_c \geq 1.5\ Hz$.

Control design



Train and test data:

$m = 1500, N = 10000, T_s = 0.01\ s$

$\delta_{f,k}$ - PRBS
r_k - Perturbed with Gaussian white noise

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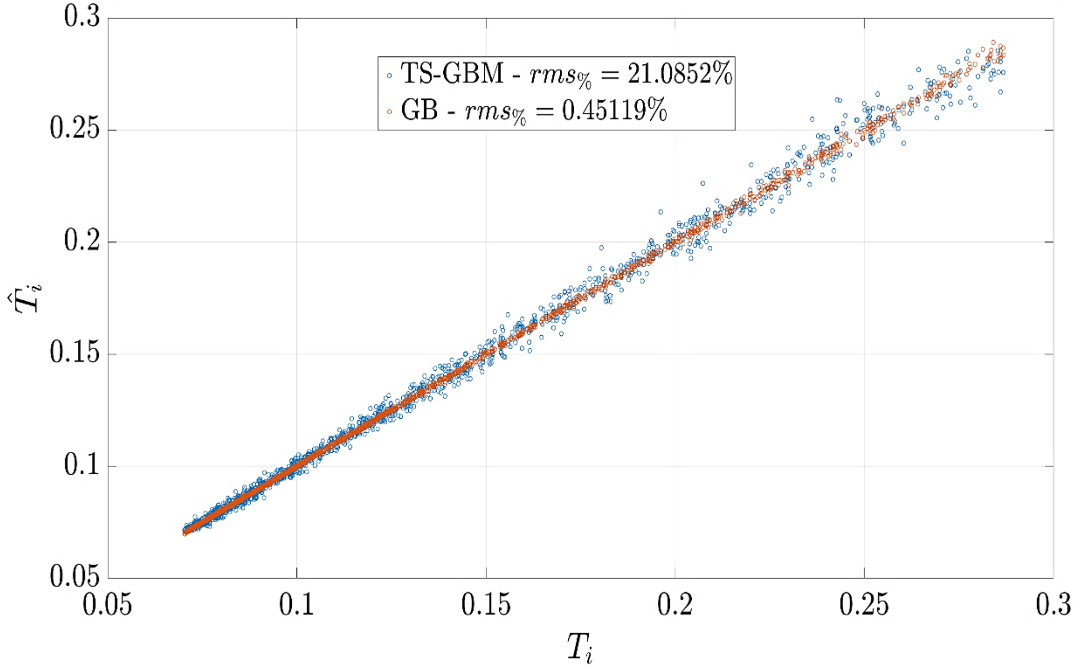
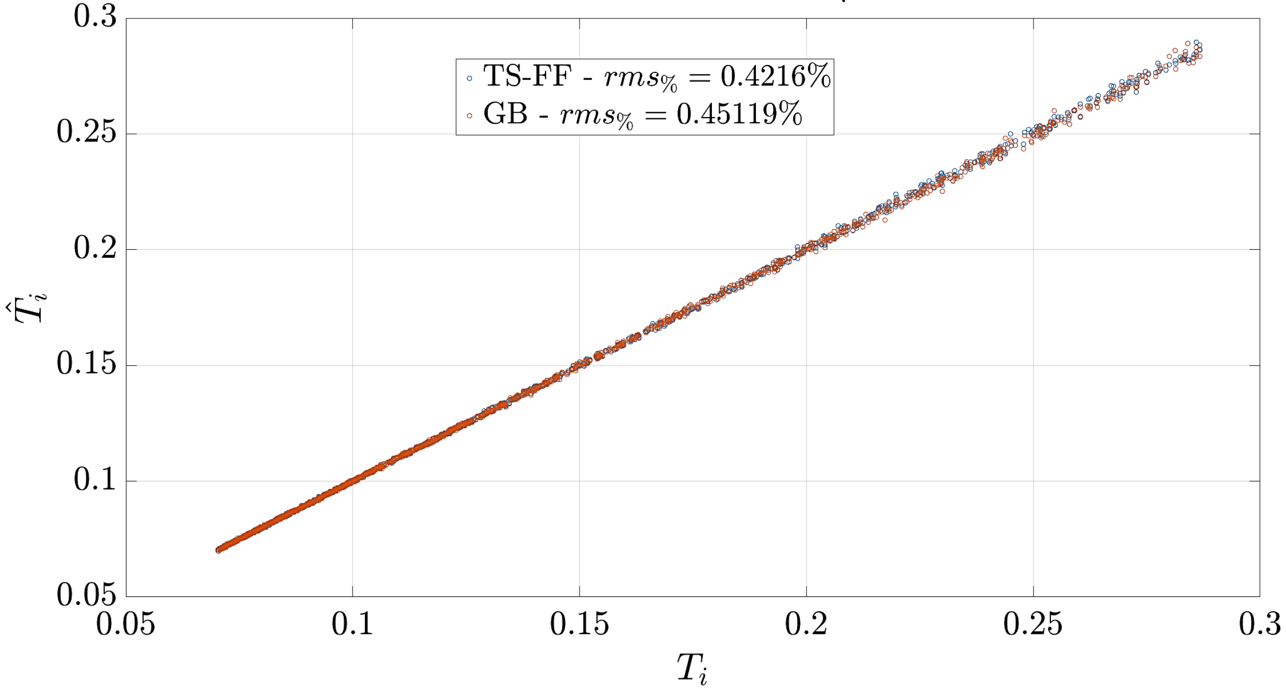
Results

T_i regression performance

$$rms\,\%\,(\nu,\hat{\nu}) = \sqrt{\frac{1}{N_s}\sum_{i=1}^{N_s}\left(100\frac{\nu_i-\hat{\nu}_i}{\nu_i}\right)^2}$$



XGBoost



FF-NN

Results

k_p regression performance

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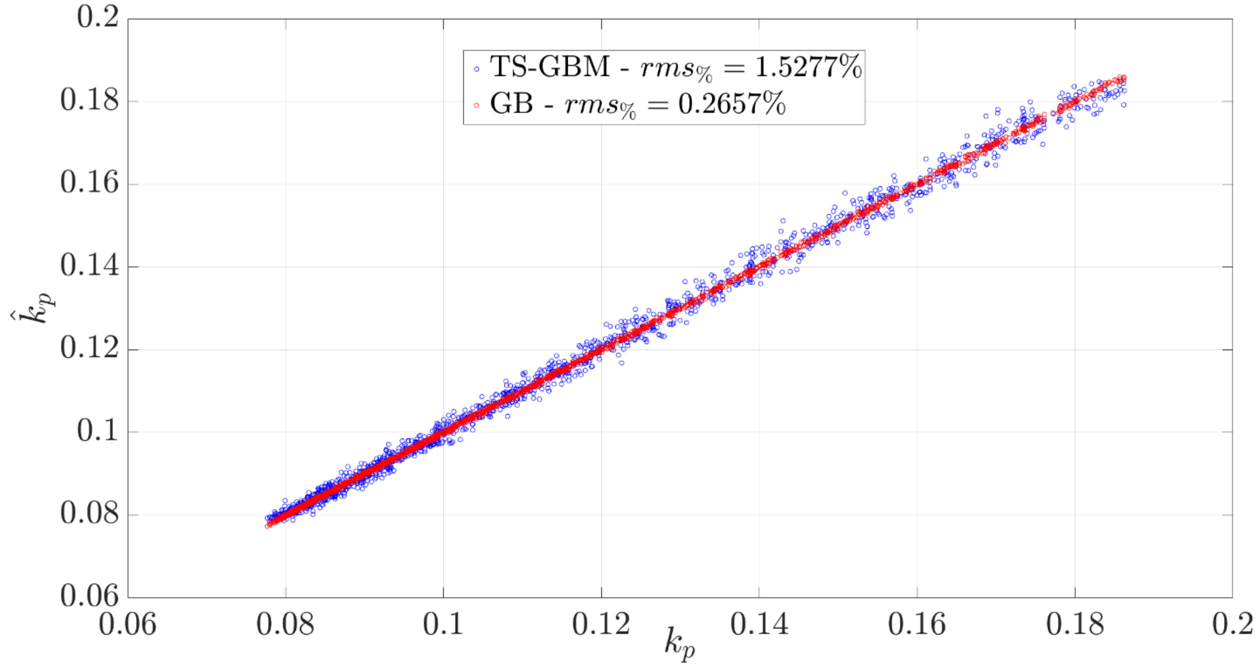

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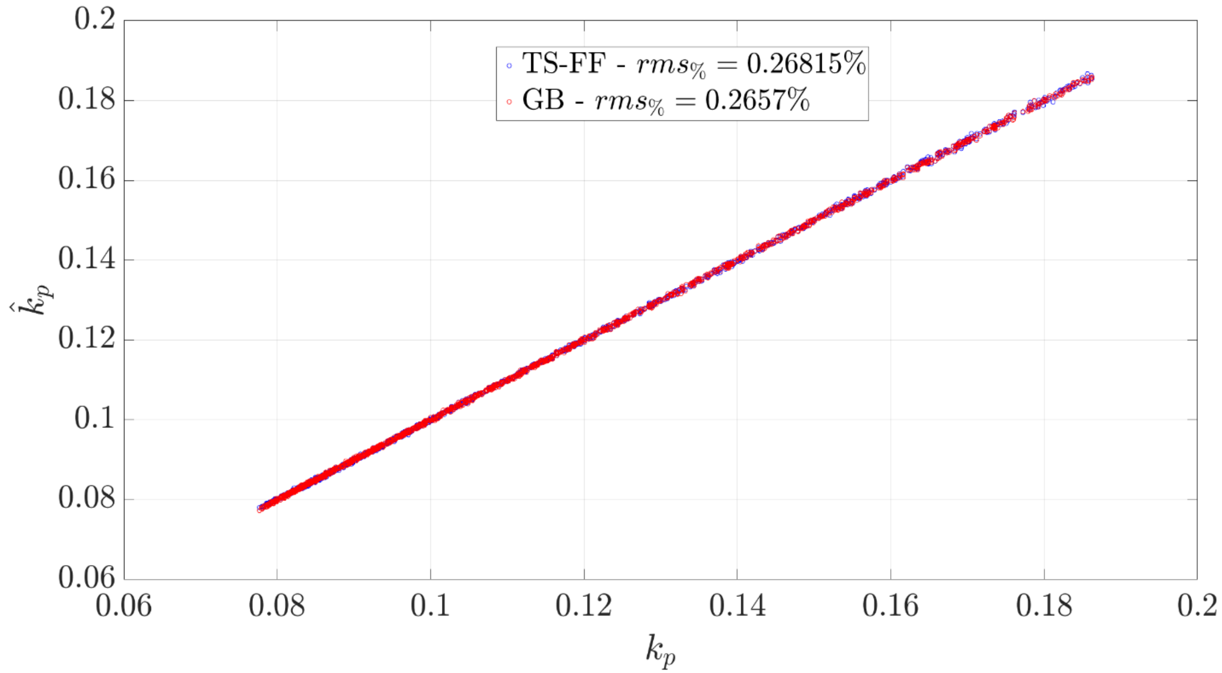


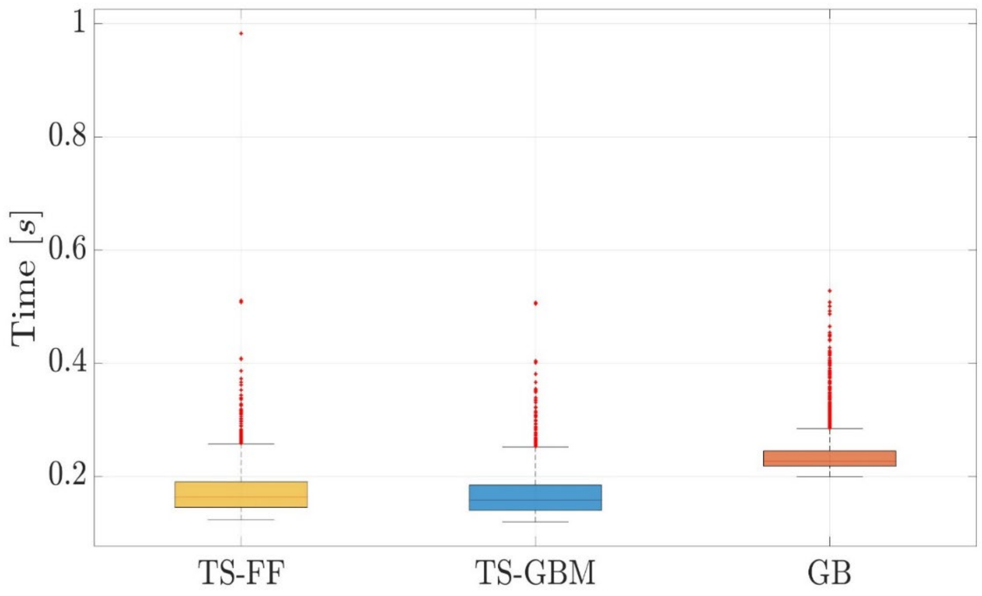
FF-NN

Results

Closed loop performance

Method	$\varphi_m[deg]$		ω_c	
	Mean	Std.	Mean	Std.
GB	59.98	0.2385	1.5	0.0037
TS-FF	59.99	0.1714	1.5	0.0032
TS-GBM	60.01	0.69	1.5	0.0143

Computation time analysis



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- First stage - ARMAX, second stage – Feed-forward neural network and XGBoost

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  - First stage helps in model reduction, while second stage acts as function approximator

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  - FF- Better accuracy in terms of controller parameters regression performance

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THANK YOU FOR YOUR ATTENTION ☺